Dielectric Constant Tensor of a Cubic Magnetic Crystal with Arbitrary Orientations of Magnetization Vector and Crystallographic Axes

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Received July 14, 1994

Abstract – Dielectric constant tensor is obtained for arbitrary orientations of the crystallographic axes and magnetization vector of a cubic magnetic crystal. The tensor is represented as the sum of two terms: the term not changing its form in the rotation of crystallographic axes and the term dependent on the rotation of the coordinate system (given for the orientations of the crystallographic axes often encountered in experiments).

INTRODUCTION

In the recent decade, substantial progress has been made in the technology of growing single-crystal ferrite-garnet films, which are of a high transparency and magneto-optical activity and in their practical applications [1, 2]. The variety of easily controlled domain structures typical of these films stimulated the development of magneto-optical studies in media with periodic domain and magnetic structures (the latter includes different structures with noncollinear spins). The interaction between laser radiation propagating in such media and magnetization results in specific static magneto-optical effects: diffraction from domains and domain boundaries [3-5], selective Bragg scattering [6, 7], and transformation of waveguide modes [8-10]. All the above indicated effects are integrated magneto-optical phenomena, which are described with the aid of the dielectric constant tensor (DCT). Thus, the first step in the solution of numerous magneto-optical problems is the construction of the DCT of a crystal for the orientation of the crystallographic axes used in the experiment. Despite the fact that the algorithms for such a construction are well known [8, 11], in many instances, the DCT is written either in the approximation of the linear magneto-optical coupling or in the principal axes [12, 13], although the contribution of the quadratic magneto-optical coupling into the phenomena under study can be either decisive or comparable with the contribution of linear coupling, as is the case, e.g., of magneto-optical waveguides [8, 14]. Below, we describe a concrete form of the DCT of a cubic crystal for an arbitrary orientation of the magnetization vector and several orientations of crystallographic axes (often encountered in practice) with respect to the film surface. The method suggested allows one to construct the DCT for any other orientations of the crystallographic axes.

DIELECTRIC CONSTANT TENSOR IN PRINCIPAL CRYSTALLOGRAPHIC AXES

It is well known that the DCT components for magnetically ordered crystals can be represented as an expansion into series in the powers of magnetization [13]:

\[
\varepsilon_{ij}(M) = \varepsilon_{ij}^{(0)} + \varepsilon_{ij}^{(1)} + \varepsilon_{ij}^{(2)} + \ldots
\]

\[
= \varepsilon_{ij}^{(0)} + \varepsilon_{ij}^{(1)} + \frac{1}{2} \varepsilon_{ij}^{(2)} M_k M_l + \ldots \quad (1)
\]

Hereafter, summation is performed over the repeating indices and \( \varepsilon_{ij}^{(0)} \) are the components of a symmetric tensor, that describe a crystal in the paramagnetic phase (at \( M = 0 \)). Introducing the directional cosines of magnetization \( \alpha_i = M_i / M \) and assuming that \( M = \text{const} \), one can write the linear and quadratic parts of the DCT in the form [13]:

\[
\varepsilon_{ij}^{(1)} = f_{ijk} \varepsilon_{kl}^{(0)} \alpha_k, \quad \varepsilon_{ij}^{(2)} = g_{ijkl} \alpha_i \alpha_j, \quad (2)
\]

where \( f_{ijk} \) are the components of the antisymmetric tensor of linear magneto-optical coefficients, which determine the circular birefringence of the magnetogyrotropic crystal and \( g_{ijkl} \) are the components of the symmetric tensor of squared magneto-optic coefficients, which determine the linear magnetic birefringence.

In cubic crystals \( \varepsilon_{ij}^{(0)} = \varepsilon_{0} \delta_{ij} \), where \( \delta_{ij} \) is the Kronecker symbol. Cubic symmetry also results in the description of linear magneto-optical coupling by a single constant, which determines the components of the tensor of linear coefficients \( f_{ijk} = if_{ijk} \), where \( e_{ijk} \) is an antisymmetric unit tensor of rank three. In the coordinate system with the \{xyz\} axes coinciding with the crystallographic \{100\} axes, the linear part of the DCT has the form

\[
\hat{\varepsilon}^{(1)} = \begin{pmatrix}
0 & \alpha_z & -\alpha_x \\
-\alpha_z & 0 & \alpha_y \\
\alpha_y & -\alpha_x & 0
\end{pmatrix} \quad (3)
\]
Quadratic magneto-optical coupling in the cubic crystals is described by three parameters $g_{11}$, $g_{12}$, and $g_{44}$ [13], which determine the form of the $\hat{g}$ tensor:

$$g_{ijk} = g_{12} \delta_{i} \delta_{k} + 2g_{44} \delta_{i} \delta_{j} + \Delta g \delta_{i} \delta_{k} \delta_{j},$$  \hspace{1cm} (4)

where $\Delta g = g_{11} - g_{12} - 2g_{44}$. From (4), it follows that these parameters are three independent components of the $\hat{g}$ tensor; $g_{11} = g_{1111}$, $g_{12} = g_{1122}$, and $g_{44} = g_{2323}$. With due regard for (2), the quadratic part of the DCT in the principal axes is

$${\hat{e}}^{(2)} = \begin{pmatrix}
g_{11} \alpha_x^2 + g_{12} (\alpha_x^2 + \alpha_y^2) & 2g_{44} \alpha_x \alpha_y & 2g_{44} \alpha_x \alpha_z \\
g_{11} \alpha_y^2 + g_{12} (\alpha_x^2 + \alpha_y^2) & 2g_{44} \alpha_y \alpha_z & 2g_{44} \alpha_y \alpha_z \\
g_{11} \alpha_z^2 + g_{12} (\alpha_x^2 + \alpha_y^2) & 2g_{44} \alpha_z \alpha_x & 2g_{44} \alpha_z \alpha_y
\end{pmatrix}. \hspace{1cm} (5)$$

DIELECTRIC CONSTANT TENSOR IN AN ARBITRARY COORDINATE SYSTEM

In the $\{x'y'z'\}$ axes of a new coordinate system, that does not coincide with the $\{100\}$ axes, the DCT components can be represented as

$$\varepsilon_{ij} = \hat{S}_{ij}' \delta_{ij} \varepsilon_{ij}, \hspace{1cm} (6)$$

where the $\hat{S}$ matrix provides the transformation of one orthonormalized basis into another one. In other words, components of the $\hat{S}$-matrix are the projections of the $e_{ij}$ unit vector of the new system onto the $e_{ij}$, unit vectors of the old system:

$$\hat{S} = \begin{pmatrix} \cos \theta & \sin \theta \sin \psi & -\sin \theta \cos \psi \\
\sin \theta \sin \varphi & \cos \varphi \cos \psi & \sin \varphi \cos \psi \\
\sin \theta \cos \varphi & -\cos \varphi \sin \psi & \cos \varphi \cos \psi\end{pmatrix}. \hspace{1cm} (7)$$

Above, we introduced three Eulerian angles: the polar angle $\varphi$ formed by the $x$ and $x'$ axes and the $\varphi$ and $\psi$ angles formed by the line of intersection of the coordinate $yz$ and $y'z'$ planes and the $y$ and $y'$ axes, respectively [11].

Unfortunately, it is difficult to use the DCT in form (6) in practice. This is associated with the fact that the directional cosines $\alpha_k$ in (6) are measured from the crystallographic $\{100\}$-axes, whereas the orientation of the $M$ vector is determined, as a rule, with respect to the symmetry axes of the specimen. Therefore, one has to pass from $\alpha_k$ to $\alpha_k'$ in (6) in order to obtain

$$\varepsilon_{ij}' = \varepsilon_{ij}(0) + f_{ij'} \alpha_k + g_{ijk'} \alpha_k \alpha_l, \hspace{1cm} (8)$$

where $\varepsilon_{ij}(0) = \varepsilon_{ij} \delta_{ij}$, and the components of the tensor of linear coefficients are

$$f_{ijk} = S_{ij} S_{jk} S_{kl} f_{ijk} = i f_{ijk}. \hspace{1cm} (9)$$

Thus, the linear part of $\varepsilon^{(1)}$ in the DCT in a new coordinate system has a form similar to (3), with $\alpha_k$ being substituted by $\alpha_k'$.

Then, the tensor of quadratic coefficients with allowance for (4) and (6) can be represented in the form:

$$g_{ijk'} = S_{ij} S_{jk} S_{kl} g_{ijk}, \hspace{1cm} (10)$$

where $G_{ijk'} = S_{ij} S_{jk} S_{kl}$, and the quadratic part of the DCT ($\hat{e}^{(2)}$) has the term

$$(\varepsilon^{(2)})_{ij} = (g_{12} \delta_{ij} \delta_{kl} + 2g_{44} \delta_{ik} \delta_{jl} + \Delta g G_{ijk'}) \alpha_k \alpha_l, \hspace{1cm} (11)$$

not changing its form in rotation. The term

$$(\varepsilon^{(2)})_{ij} = \Delta g G_{ijk'} \alpha_k \alpha_l, \hspace{1cm} (12)$$

is dependent on the rotation of the coordinate system (in other words, the $G_{ijk'}$ coefficients are the functions of the Eulerian angles).
One can now write the DCT terms without changing their form upon the rotation transformations:

\[
\hat{\varepsilon}^{(0)\prime} + \hat{\varepsilon}^{(1)\prime} = \begin{pmatrix}
\varepsilon_0 & \text{if } \alpha_x = -i \alpha_y \\
-\varepsilon_0 & \text{if } \alpha_x = i \alpha_y \\
\text{if } \alpha_x = -i \alpha_y & \varepsilon_0
\end{pmatrix},
\]

Equation (13)

\[
g^{(2)\prime}_i = \begin{pmatrix}
g_{12} + 2g_{44}\alpha_2^2 & 2g_{44}\alpha_2\alpha_y & 2g_{44}\alpha_y \alpha_z \\
g_{12} + 2g_{44}\alpha_2\alpha_y & g_{12} + 2g_{44}\alpha_2^3 & 2g_{44}\alpha_2\alpha_z \\
g_{12} + 2g_{44}\alpha_y \alpha_z & 2g_{44}\alpha_y \alpha_z & g_{12} + 2g_{44}\alpha_2^3
\end{pmatrix}.
\]

PARTICULAR ORIENTATIONS OF CRYSTALLOGRAPHIC AXES

Now determine the term \( \hat{\varepsilon}_i^{(2)\prime} \) for orientations of the crystallographic axes often encountered in practice.

Using (13) and (15), one can write the nonzero DCT components for the widespread case of magnetization-vector orientation along the \([111]\)-axis (i.e., \( \alpha_x = 1, \alpha_y = 0, \) and \( \alpha_z = 0 \)) often encountered in epitaxial films anisotropic along the normal direction:

\[
\varepsilon_{x'x'} = \varepsilon_0 + g_{12} + 2g_{44} + \Delta g/3,
\]

\[
\varepsilon_{y'y'} = \varepsilon_{z'z'} = \varepsilon_0 + g_{12} + \Delta g/3,
\]

\[
\varepsilon_{x'z'} = -\varepsilon_{x'z'} = i\varepsilon_\sigma.
\]

Equation (16)

B. In the waveguide structures, one often uses epitaxial films with surfaces coinciding with the crystallographic \([\bar{1} 1 0]\) plane. In this case, the \([x'y'z']\)-axes can be related to the crystallographic \([1 1 0]\), \([1 0 0]\), and \([0 0 1]\) axes. Thus, the \( \hat{S} \) matrix has the form

\[
\hat{S} = \begin{pmatrix}
1/3 & 1/3 & 1/3 \\
1/3 & 1/2 & 1/6 \\
0 & 0 & 0
\end{pmatrix}
\]

Equation (17)

The nonzero components of the \( \hat{G} \) tensor introduced by relationship (10) are: \( G_{x'x'x'} = G_{y'y'y'} = 1/2 \), \( G_{z'z'z'} = 1 \), and \( G_{x'y'y'} = 1/2 \), where \( \sigma \) denotes an arbitrary permutation of the \((x'y'z')\) subscripts. Knowing \( \hat{G} \), one can readily obtain all the components of the \( \hat{\varepsilon}_i^{(2)\prime} \) tensor. We can now write all the nonzero components of the DCT tensor for \( \alpha_x = \alpha_y = 0 \) and \( \alpha_z = 1 \):

\[
\varepsilon_{x'x'} = \varepsilon_{y'y'} = \varepsilon_{z'z'} = \varepsilon_0 + g_{44},
\]

\[
\varepsilon_{x'z'} = \varepsilon_{z'x'} = i\varepsilon_\sigma.
\]

Equation (18)

If the magnetization vector is aligned in the \([110]\) direction in the film plane, i.e., if \( \alpha_x = \alpha_y = 0 \) and \( \alpha_z = 1 \), then

\[
\varepsilon_{x'x'} = \varepsilon_{y'y'} = \varepsilon_{z'z'} = \varepsilon_0 + g_{12} + 2g_{44} + \Delta g/2,
\]

\[
\varepsilon_{x'z'} = \varepsilon_{z'x'} = \varepsilon_0 + \Delta g,
\]

Equation (19)

C. In some cases, the growing film surface is the \([2 2 1]\) plane. Assuming that the \([x'y'z']\) axes coincide

\[
\hat{S} = \begin{pmatrix}
1/\sqrt{2} & -1/\sqrt{2} & 0 \\
1/\sqrt{2} & 1/\sqrt{2} & 0 \\
0 & 0 & 1/3
\end{pmatrix}
\]

Equation (15)
with the crystallographic [211], [01\bar{1}], and [\bar{1}11] axes, we obtain:

$$
\hat{S} = \begin{pmatrix}
2/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{6} \\
1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\
-1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3}
\end{pmatrix}.
$$

(20)

In accordance with (12), the $\hat{G}$ matrix has the form

$$
\hat{G} = \begin{pmatrix}
1/2 & 1/6 & 1/3 & 0 & -1/3 & \sqrt{2} & 0 \\
1/6 & 1/2 & 1/3 & 0 & 1/3 & \sqrt{2} & 0 \\
1/3 & 1/3 & 1/3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/3 & 0 & 1/3 & \sqrt{2} \\
-1/3 & \sqrt{2} & 1/3 & \sqrt{2} & 0 & 1/3 & \sqrt{2} & 1/3 & 0 \\
0 & 0 & 0 & 1/3 & \sqrt{2} & 0 & 1/6
\end{pmatrix}.
$$

(21)

It should be noted that in the process of film growth, the crystallographic axes normal to the film surface is fixed. Two other orthogonal axes lying in the film plane do not necessarily coincide with the symmetry axes of the specimen. Thus, the $x'$ axis coincides with the [111] normal, whereas the angles formed by the $y'$ and [11\bar{2}] and by the $z'$ and [\bar{1}10] axes are equal to $\beta$ (see the figure). Then, in order to determine the $\hat{\varepsilon}$ tensor, one has to substitute the $\hat{G}$ matrix by $\hat{P}^{-1}\hat{G}\hat{P}$, where the $\hat{P}$ matrix provides the rotation of the angle $\beta$ around the $x'$ axis:

$$
\hat{P} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \cos^2\beta & \sin^2\beta & \sin 2\beta & 0 & 0 \\
0 & \sin^2\beta & \cos^2\beta & -\sin 2\beta & 0 & 0 \\
0 & -\sin 2\beta & \sin 2\beta & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & \cos\beta & -\sin\beta \\
0 & 0 & 0 & 0 & \sin\beta & \cos\beta
\end{pmatrix}.
$$

(22)

The above expressions allow one to write the DCT for ferrite-garnet films in almost all the experimental situations.

In conclusion, it should be noted that a mismatch of the lattice parameters of a growing single-crystal ferrite-garnet films and the substrate results in tetrahedral or orthorhombic distortions of the film structure. These distortions change the form of the DCT and thus also change the magneto-optical effects in grown films. The relations between the DCT and the deviation from cubic symmetry will be considered in another article.

REFERENCES


