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Reflection and Transmission of Light in Structures with Incoherent Anisotropic Layers

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Abstract—On the basis of four-component coherence vectors and their transformation matrices, the reflection and transmission of light in plain layered anisotropic structures containing thin coherent and thick incoherent layers is analyzed.

INTRODUCTION

Nowadays, planar layered structures consisting of thin layers whose thickness is exactly specified and uniform throughout the structure surface and thick layers whose thickness irregularly changes from point to point and is determined as an average quantity are extensively used for the production of various optical devices. Different theoretical methods for the description of optical properties of the structures containing layers of the first type (coherent layers) were developed in the literature [1–3]. Considerable recent attention has been devoted to the analysis of optical properties of the structures containing layers of the second type (incoherent layers). In [4, 5], a dielectric layer with a random change of optical thickness and a Fabry–Perot interferometer made on its basis were considered. In actual practice, layered structures, as a rule, consist of a combination of layers of both types [6]. In [7], the effect of multiple reflection of light in a thick substrate on the reflectivity of a surface layer found on one of its interfaces was analyzed. In [8], Mueller matrices for a system of thin and thick isotropic layers were obtained. The case where a structure contains anisotropic layers is more complicated. An example of such a structure is a thin anisotropic film on a thick dielectric substrate, which is extensively used [9].

In this paper, we use four-component coherence vectors and their transformation matrices and obtain on their basis the matrices describing reflection and transmission of light traveling through multilayer anisotropic systems containing coherent and incoherent layers. For a structure representing a thin magnetogyrotropic film on a thick substrate, dependences of intensity and polarization characteristics of the reflected and transmitted waves on its parameters are constructed.

INCOHERENT LAYERED STRUCTURE

In many cases, multilayer optical structures contain layers whose thickness considerably exceeds the wavelength of radiation traveling through them and is therefore irregular across the surface of a sample illuminated with a light beam incident on it. Because of this irregularity, a totally coherent polarized wave interacting with a structure becomes spatially incoherent and only partially polarized. To analyze the state of the field in the reflected wave and the wave passed through such layers, let us use the method in which the field is described by a four-component coherence vector and its transformation is described by the corresponding 4×4 matrix M:

$$\mathbf{Y} = \begin{pmatrix} Y_{xx} \\ Y_{xy} \\ Y_{yx} \\ Y_{yy} \end{pmatrix} = \begin{pmatrix} \langle E_x E_x^* \rangle \\ \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle \\ \langle E_y E_y^* \rangle \end{pmatrix}, \quad \mathbf{Y}^{(r,t)} = M^{(r,t)} \mathbf{Y}^{(i)}, \quad (1)$$

where four components of the coherence vector are determined by the averaged products of the corresponding components of the electric field, with averaging being carried out over the cross section of the beam, and completely characterize its state [10]. For a coherent optical system, one can find the matrix M from the conventional 2×2 reflection and transmission matrices which give the relationship between the x and y components of the electric field in the incident (i) and reflected (r) or transmitted (t) waves:

$$\begin{pmatrix} E_x^{(r)} \\ E_y^{(r)} \end{pmatrix} = \begin{pmatrix} r_{xx} & r_{xy} \\ r_{yx} & r_{yy} \end{pmatrix} \begin{pmatrix} E_x^{(i)} \\ E_y^{(i)} \end{pmatrix},$$
$$\begin{pmatrix} E_x^{(t)} \\ E_y^{(t)} \end{pmatrix} = \begin{pmatrix} t_{xx} & t_{xy} \\ t_{yx} & t_{yy} \end{pmatrix} \begin{pmatrix} E_x^{(i)} \\ E_y^{(i)} \end{pmatrix}. \quad (2)$$

Let the elements of the transformation matrix in relations (2) be known ($a_{\alpha\beta}$, where $\alpha, \beta = x, y$). In this case, the corresponding transformation matrix for the coherence vector has the form

$$M(a) = \frac{1}{2}\begin{pmatrix} a_{xx}a_{xx}^* & a_{xx}a_{xy}^* & a_{xy}a_{xx}^* & a_{xy}a_{xy}^* \\ a_{xx}a_{yx}^* & a_{xx}a_{yy}^* & a_{xy}a_{yx}^* & a_{xy}a_{yy}^* \\ a_{yx}a_{xx}^* & a_{yx}a_{xy}^* & a_{yy}a_{xx}^* & a_{yy}a_{xy}^* \\ a_{yx}a_{yx}^* & a_{yx}a_{yy}^* & a_{yy}a_{yx}^* & a_{yy}a_{yy}^* \end{pmatrix}. \quad (3)$$

To find optical characteristics of multilayer structures containing layers of both types, first let us consider a thick layer interacting with light. Later on, it may be included into a more general structure as a subsystem. Let a plane monochromatic light wave be incident from the first medium at a certain angle onto a structure consisting of two (j = 1, 2) plane layered systems, which are separated by a thick anisotropic layer, and then travels through a third medium. Let us assume that the 4×4 reflection and transmission matrices of the systems found in the given environment of the media are known: R'_j, R''_j, T'_j, T''_j, and T'_q, T''_q are the reflection and transmission matrices of the layered systems and the separating layer for the forward (') and backward ('') directions.

By using the method of multiple reflections, we find the 4×4 reflection and transmission matrices R and T for the whole system. In the first system, the incident wave $\mathbf{Y}_0$ is divided into two plane waves, namely, the reflected wave and the wave that enters the separating layer. In turn, the transmitted wave traveling through the layer is transformed and incident on the second layer, where it is divided into two plane waves again. One of them travels in the forward direction and the other one is reflected backward into the layer. The process of division of a wave remaining in a layer continues. As a result, the superposition of all reflected waves forms the wave whose coherence vector is given by the expression

$$\mathbf{Y}^{(r)} = \left[R'_1 + T''_1T''_qR'_2\sum_{m=0}^{\infty}(T'_qR''_1T''_qR'_2)^m T'_qT'_1\right]\mathbf{Y}^{(i)}. \quad (4)$$

By making summation in (4), we obtain the reflection matrix in the case of a wave incident on a layered structure in the forward direction:

$$R' = R'_1 + T''_1T''_qR'_2(I - T'_qR''_1T''_qR'_2)^{-1}T'_qT'_1, \quad (5)$$

where I is the unit matrix. In the same way, one can obtain the expression for the coherence vector of the transmitted wave and the transmission matrix of a layered system for the forward direction:

$$\mathbf{Y}^{(t)} = T'\mathbf{Y}^{(0)}, \quad T' = T'_2(I - T'_qR''_1T''_qR'_2)^{-1}T'_qT'_1. \quad (6)$$

The matrices R'' and T'' for the backward direction can be obtained from R' and T' by way of replacing the superscripts (') and ('') with "1" and "2".

The reflection and transmission matrices for the structures containing several thick layers can be obtained by sequential application of formulas (5) and (6) and finding, in a recurrent way, the corresponding matrices for successively larger subsystems, starting with one of the thick layers.

By using matrices (5) and (6), let us find optical characteristics of the reflected and transmitted waves for a linearly polarized wave with p or s polarization incident at the angle θ_1, in the case where the structure is placed between two isotropic media. Let M be the matrix describing the transformation of the wave caused by its interaction with the structure, i.e., one of the 4×4 matrices R or T. By analogy with (1), let us introduce the coherence vectors expressed in terms of the field components E_p and E_s:

$$\mathbf{G} = \begin{pmatrix} G_{pp} \\ G_{ps} \\ G_{sp} \\ G_{ss} \end{pmatrix} = \begin{pmatrix} \langle E_pE_p^*\rangle \\ \langle E_pE_s^*\rangle \\ \langle E_sE_p^*\rangle \\ \langle E_sE_s^*\rangle \end{pmatrix} = \begin{pmatrix} Y_{xx}/\cos^2\theta_j \\ Y_{xy}/\cos\theta_j \\ Y_{yx}/\cos\theta_j \\ Y_{yy} \end{pmatrix}, \quad (7)$$

where $j = 1$ for the incident wave, and $j = 2$ for the transmitted wave. Moreover, we took into account that $E_x = E_p\cos\theta_j$, and $E_y = E_s$. In the case of p- and s-polarized waves incident on the structure, we have $\mathbf{Y}_x^{(0)} = (\cos^2\theta_1, 0, 0, 0)$, and $\mathbf{Y}_y^{(0)} = (0, 0, 0, 1)$, and the $\mathbf{G}$ vectors for the output wave $\mathbf{Y}_{x,y} = M\mathbf{Y}_{x,y}^{(0)}$ have the form

$$\mathbf{G}_p = \cos^2\theta_1\begin{pmatrix} M_{11}/\cos^2\theta_j \\ M_{21}/\cos\theta_j \\ M_{31}/\cos\theta_j \\ M_{41} \end{pmatrix},$$
$$\mathbf{G}_s = \begin{pmatrix} M_{14}/\cos^2\theta_j \\ M_{24}/\cos\theta_j \\ M_{34}/\cos\theta_j \\ M_{44} \end{pmatrix}. \quad (8)$$

Let us expand the field in the resulting wave whose intensity is equal to $G_{pp} + G_{ss}$ in mutually independent polarized and nonpolarized parts and represent the degree of polarization of the initial wave in the form

$$P = \left(1 - 4\frac{G_{pp}G_{ss} - G_{ps}G_{sp}}{(G_{pp} + G_{ss})^2}\right)^{1/2}. \qquad (9)$$

The shape and the orientation of the polarization ellipse, which is associated with the polarized part of the wave, are specified by the ellipticity χ and the rotation of the plane of polarization ψ, which are determined by the expressions

$$\tan 2\chi = i(G_{sp} - G_{ps})/\sqrt{(G_{pp} - G_{ss})^2 + (G_{ps} + G_{sp})^2},$$
$$\tan 2\psi = (G_{ps} + G_{sp})/(G_{pp} - G_{ss}). \qquad (10)$$

MAGNETOGYROTROPIC FILM ON A SUBSTRATE

To illustrate the application of the formalism presented above, let us consider a layered structure in the form of an anisotropic film on a substrate, which is extensively used in practice. Let us assume that the film represents a thin magnetogyrotropic layer whose magnetization is perpendicular to the interfaces and introduce the following notation: ρ_i and τ_i are the Fresnel reflectivities and transmittances for the air–gyrotropic layer, gyrotropic layer–substrate, and substrate–air interfaces (i = 1, 2, 3), $\varphi = d_f\varphi_f$, where φ_f is the specific Faraday rotation, n_j is the refractive index, d_j are the film and substrate thicknesses ($j = f, q$), and k_0 is the wave number in vacuum. Taking into account the first-order effects with respect to the magneto-optical parameter, one can write the transmission and reflection matrices of the gyrotropic layer ($\hat{r}$ and $\hat{t}$) and the substrate ($\hat{t}_q$) in the case of normal incidence in the form

$$\hat{r}' = \rho_1\hat{I} + \frac{1}{D}(n_f/n_1)\rho_2\tau_1^2\exp(i\beta_f)\begin{pmatrix} p & a \\ -a & p \end{pmatrix},$$

$$\hat{r}'' = \rho_2\hat{I} - \frac{1}{D}(n_2/n_f)\rho_1\tau_2^2\exp(i\beta_f)\begin{pmatrix} p & a \\ -a & p \end{pmatrix},$$

$$\hat{t}' = \hat{t}'' = \frac{1}{D}\tau_1\tau_2\exp(i\beta_f)\begin{pmatrix} q & b \\ -b & q \end{pmatrix}, \qquad (11)$$

$$\hat{t}_q' = \hat{t}_q'' = \hat{I}\exp(i\beta_q),$$

where

$$D = 1 + 2\rho_1\rho_2\cos(2\varphi)\exp(2i\beta_f) + \rho_1^2\rho_2^2\exp(4i\beta_f),$$

$$p = \cos 2\varphi + \rho_1\rho_2\exp(2i\beta_f), \quad q = (1 + \rho_1\rho_2)\cos\varphi,$$

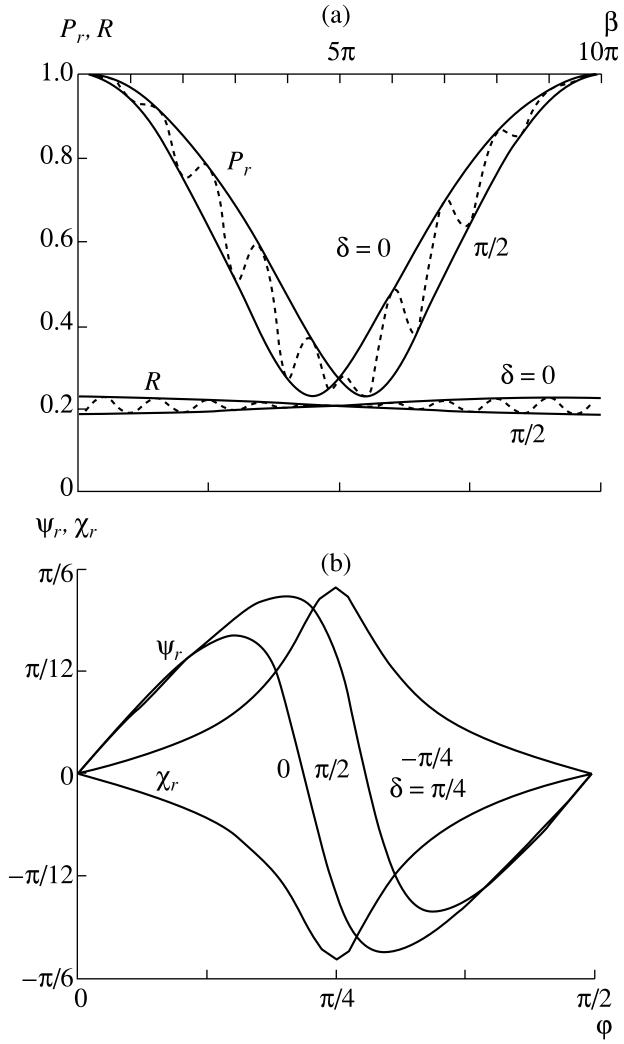

Fig. 1. (a) The reflectivity R and the degree of polarization P_r and (b) the rotation of the plane of polarization ψ_r and the ellipticity χ_r for the reflected wave in the case of an incoherent substrate as functions of the angle of Faraday rotation in a film (solid curves) for two values of the parameter δ and of the normalized film thickness (dashed curves).

$$a = \sin 2\varphi, \quad b = (1 - \rho_1\rho_2)\sin\varphi,$$
$$\beta_f = k_0 d_f n_f, \quad \beta_q = k_0 d_q n_q.$$

Starting from the coherent matrices obtained above and using (3), one can obtain the transformation matrices for the vector **Y** for the gyrotropic layer, the substrate, and the substrate–air interface. In this case, the corresponding 2×2 matrices change to the 4×4 matrices:

$$\hat{r}' \longrightarrow R_1', \quad \hat{r}'' \longrightarrow R_1'', \quad \hat{t}' \longrightarrow T_1', \quad \hat{t}'' \longrightarrow T_1'',$$

$$\hat{r}_3' \longrightarrow R_3', \quad \hat{r}_3'' \longrightarrow R_3'', \quad \hat{t}_3' \longrightarrow T_3', \quad \hat{t}_3'' \longrightarrow T_3'',$$

$$T_q' = T_q'' = I.$$

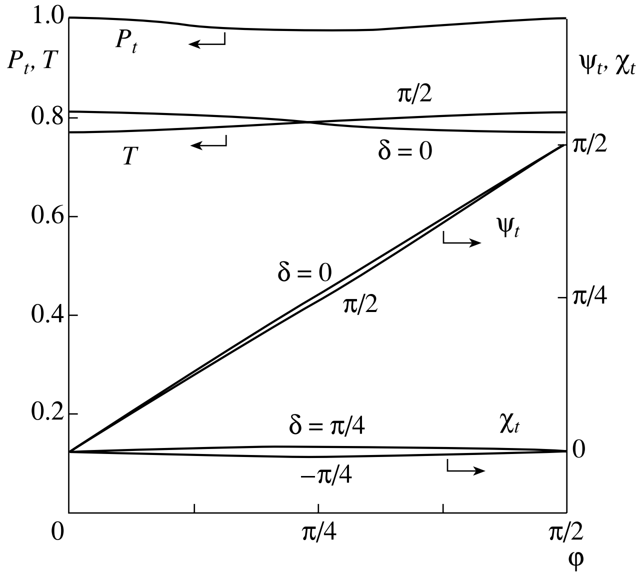

Fig. 2. Dependences of intensity and polarization characteristics of the transmitted wave in the case of an incoherent substrate on the angle of Faraday rotation in a film for two values of the parameter δ.

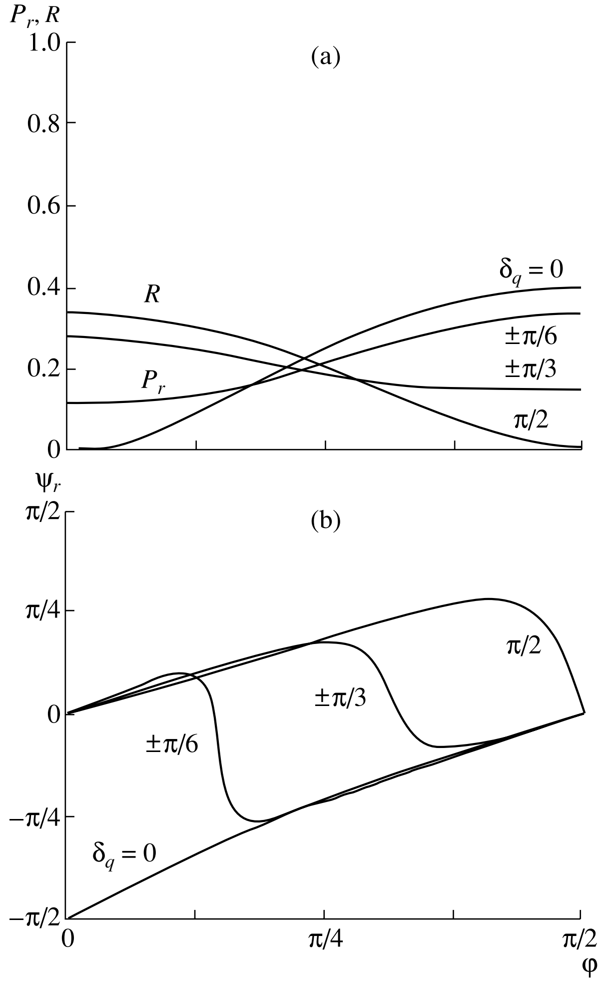

Fig. 3. (a) The reflectivity R and (b) the rotation of the plane of polarization ψ_r for the reflected wave in the case of a coherent substrate as functions of the angle of Faraday rotation in a film for several values of the parameter δ_q.

As a result, the matrices of the whole structure, which determine the transformation of the fields in the reflected and transmitted waves, take the form (for the forward propagation)

$$R' = R'_1 + T''_1 R'_3 (I - R''_1 R'_3)^{-1} T'_1, \qquad T' = T'_3 (I - R''_1 R'_3)^{-1} T'_1. \tag{12}$$

By using matrices (12) and the coherence vectors of the wave incident on the structure, one can find optical characteristics of the structure, in particular, the reflectivities and transmittances, the degree of polarization, the rotation of the plane of polarization, and the ellipticity of the transmitted and reflected waves.

Let us use the relations obtained above to carry out a numerical analysis of optical characteristics of a concrete layered magnetogyrotropic structure, in the case of normal incidence of a light wave on this structure. Let the refractive indices of a thin magnetic film and a thick nonmagnetic substrate be $n_f = 2.11$ and $n_q = 1.95$, respectively, which corresponds to parameters of epitaxial garnet-type ferrite films and substrates made of gadolinium gallium garnet. An analysis of relations (11) and (12) shows that all optical characteristics of the reflected and transmitted waves have periodic dependences with period π on the angle of Faraday rotation φ and the parameter β_f. The intensity and the degree of polarization are even functions of the angle φ, whereas the rotation of the plane of polarization and the ellipticity are odd functions.

Figure 1 presents the dependences of (a) the reflectivity R and the degree of polarization P_r and (b) the rotation of the plane of polarization ψ_r and the ellipticity χ_r for the reflected wave on the angle of Faraday rotation φ (solid curves) for two limiting values of the parameter δ, which specifies the normalized film thickness by the relation $\beta_f = n\pi + \delta$ ($n = 1, 2, \ldots$). For δ different from the limiting values, the curves determining the corresponding dependence on the angle φ lie between the curves plotted in the figures. Moreover, Fig. 1a presents the dependences of R and P_r on the normalized film thickness (dashed curves) for $\varphi_f = k_0 n_f/20$. Figure 2 presents the dependences of intensity and polarization characteristics of the wave transmitted through the structure. The dependences of optical characteristics for the reflected wave and especially for the transmitted wave on the film thickness for a given value of φ are weak, owing to the fact that the interaction of light with the film–substrate interface is weak because of a small difference of refractive indices. The reflected and transmitted waves are totally polarized at the points

$\varphi = m\pi/2$ ($m = 0, 1, 2, \ldots$), and the weakest polarization is obtained near the points $\varphi = (m + 1/2)\pi/2$. The maximum angle of rotation of the plane of polarization for the reflected wave is slightly greater than $\pi/10$.

For comparison, Fig. 3 presents the dependences of optical characteristics of the reflected wave on the angle φ for the case where a substrate represents a coherent layer, for $\delta = 0$ and several values of the parameter δ_q ($\beta_q = n\pi + \delta_q$). For a film with $\delta \neq 0$, the form of the dependences is determined by the total structure thickness and; consequently, the summary quantity $\delta + \delta_q$ because the reflection at the film–substrate interface is weak. In view of this fact, one may approximately state that the curve with the substrate parameter equal to $\delta + \delta_q$ corresponds to a structure with arbitrary values of δ and δ_q. Note the following distinctions between coherent and incoherent propagation of a wave in a magnetogyrotropic structure. In the incoherent case, the substrate thickness is not exactly determined; therefore parameters of the reflected and transmitted waves are independent of this thickness and depend on the film thickness only slightly. In this case, the governing factor is the angle of Faraday rotation in the film. In the case of coherent propagation, the transmitted and reflected waves are totally polarized, and their characteristics strongly depend both on the film and substrate thicknesses and on the angle of Faraday rotation.

CONCLUSIONS

The results obtained in the paper can be used for the study of optical characteristics of various systems consisting of plane-parallel coherent and incoherent anisotropic layers.

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