Possibility theory, probability theory and multiple-valued logics: A clarification

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Abstract:
There has been a long-lasting misunderstanding in the literature of artificial intelligence and uncertainty modeling, regarding the role of fuzzy set theory and many-valued logics. The recurring question is that of the mathematical and pragmatic meaningfulness of a compositional calculus and the validity of the excluded middle law. This confusion pervades the early developments of probabilistic logic, despite early warnings of some philosophers of probability. This talk tries to clarify this situation. It emphasizes three main points. First, it suggests that the root of the controversies lies in the unfortunate confusion between degrees of belief and what logicians call "degrees of truth". The latter are usually compositional, while the former cannot be so. This claim is first illustrated by laying bare the non-compositional belief representation embedded in the standard propositional calculus. It turns out to be an all-or-nothing version of possibility theory. This framework is then extended to discuss the case of fuzzy logic versus graded possibility theory. Next, it is demonstrated that any belief representation where compositionality is taken for granted is bound to at worst collapse to a Boolean truth assignment and at best to a poorly expressive tool. Lastly, some claims pertaining to an alleged compositionality of possibility theory are refuted, thus clarifying a pervasive confusion between possibility theory axioms and fuzzy set basic connectives. Lastly, some connections between numerical possibility and probability will be outlined based on the fact that possibility distributions may encode families of probabilities. This talk is partially based on a paper with Henri Prade, to appear in the Annals of Mathematics for Artificial Intelligence