Individual Project 1

Model of Water Quality Control in Schweriner Lake

Polluted water enters Schweriner lake from a recently built factory at a constant rate N. A mathematical model of concentration of pollution has been developed under certain assumptions including the following:

- the upper levels of water are mixed in all directions
- change in mass of pollution is equal to the difference between the mass of the entering pollution and the mass of the pollution which is being decomposed
- the rate of decomposition of pollution is constant
- decomposition of pollution takes place due to biological, chemical and physical processes and/or exchange with the deeper levels of water.

Under the assumptions the equation of the balance of mass of pollution on any interval of time Δt can be written in the form:

$$V\Delta C_N = N\Delta t - QC_N\Delta t - KVC_N\Delta t, \qquad (1)$$

where V – volume of the upper levels (constant), Q – water consumption rate (constant), $C_N = C_N(t)$ - concentration of pollution at time t, K – decomposition rate (constant).

Questions:

1) Set up a differential equation for the concentration of pollution from (1) by dividing both sides of equation (1) by Δt and taking a limit when $\Delta t \to 0$.

2) Solve that differential equation to find the concentration of pollution as a function of time provided that the initial concentration of pollution was zero.

3) Determine the equilibrium concentration of pollution C_{Ne} (that is the concentration when $t \to \infty$ or $\frac{dC_N}{dt} = 0$).

4) Determine time needed to reach p portion ($p = C_N(t)/C_{Ne}$) of the equilibrium concentration. Will it take more time to reach p portion of the equilibrium concentration in case when there is no decomposition of pollution?

5) Set up a differential equation for the concentration of pollution from the differential equation in question 1 in case when the initial amount of pollution entered the lake was C_o and after that pollution is not coming to the lake anymore (that is N = 0).

6) Solve the differential equation from question 5.

7) Determine time needed to reach $1 - p$ portion ($C(t)/C_0 = 1 - p$) of the initial concentration C_0.