

Individual Project 5

Populations of Birds and Insects on Pöel Island

Populations of birds and insects on Pöel Island are modelled by the equations:

$$\begin{aligned}\frac{dx}{dt} &= 0.4x - 0.002xy \\ \frac{dy}{dt} &= -0.2y + 0.000008xy\end{aligned}\tag{1}$$

Questions:

- Which of the variables, x or y , represents the bird population and which represents the insect population? Explain.
- Find the equilibrium solutions and explain their significance.
- Find an expression for dy/dx and solve this separable differential equation with pen and paper.
- Use Matlab (ppplane) to draw the phase trajectory corresponding to initial population of 100 birds and 40,000 insects. Use the phase trajectory to describe how both populations change with respect to time.
- Use part d) to make rough sketches of the bird and insect populations as functions of time. How are these graphs related to each other?

Suppose equations (1) are replaced by the equations:

$$\begin{aligned}\frac{dx}{dt} &= 0.4x(1 - 0.000001x) - 0.002xy \\ \frac{dy}{dt} &= -0.2y + 0.000008xy\end{aligned}\tag{2}$$

- According to equations (2), what happens to the insect population in the absence of birds?
- Use Matlab (ppplane) to draw the phase trajectory for equations (2) corresponding to initial population of 100 birds and 40,000 insects. Use the phase trajectory to describe how both populations change with respect to time.