

Maple Project II

Slope Fields: Viewing Solution Curves

Introduction.

Each time we specify an initial condition

$$y(x_0) = y_0$$

for the solution of an ODE

$$y' = f(x, y),$$

the solution curve (graph of the solution) should pass through the point (x_0, y_0) and should have slope $f(x_0, y_0)$ there. We can picture these slopes graphically by drawing short line segments of slope $f(x, y)$ at selected points (x, y) in the region of the xy -plane that constitutes the domain of the function f . Each segment has the same slope as the solution curve through (x, y) and so is tangent to the curve there. The resulting picture is called a **slope field** (or **direction field**) and gives a visualization of the general shape of the solution curves. Maple has the ability to easily plot slope fields. To do this we first need to load the Maple package **DEtools**.

```
> with(DEtools):
```

Suppose we want to investigate the solutions to the differential equation

$$y' = y - x$$

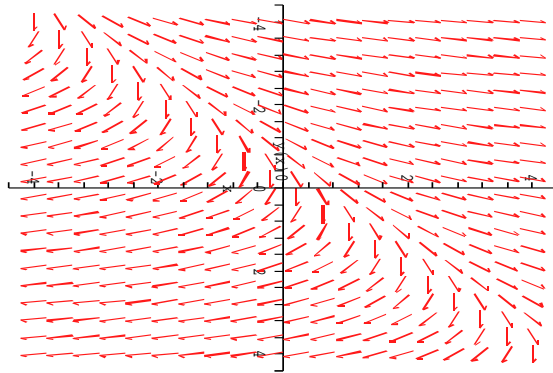
by first examining its slope field. The first step is to rewrite our ODE in maple format:

```
> ode1:=diff(y(x), x)=y(x) - x;
```

$$ode1 := \frac{d}{dx}y(x) = y(x) - x$$

To plot the slope field on $[-4, 4] \times [-4, 4]$, use “dfieldplot” as follows:

```
> dfieldplot(ode1, y(x), x=-4..4, y=-4..4);
```

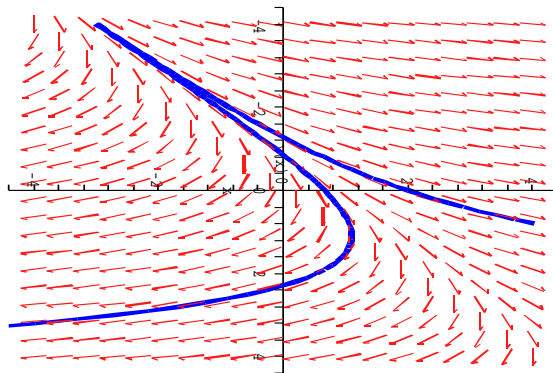


To input initial conditions $y(0) = 2/3$ and $y(0) = 2$, use

```
> init1:={ [0, 2/3], [0, 2] }:
```

Use “DEplot” to generate solution curves corresponding to the initial conditions given.

```
> DEplot(ode1, y(x), x=-4..4, y=-4..4, init1, stepsize=0.1, linecolor=blue);
```



In maple you can also derive the general solution to the ODE by using “dsolve”. For example

```
> dsolve(ode1);
```

$$y(x) = x + 1 + e^x _C1$$

It is the general solution to $y' = y - x$, the particular solution with $y(0) = 2/3$ is given by:

```
> dsolve(ode1, y(0)=2/3);  
y(x) = x + 1 - 1/3 e^x
```

Homework:

In the exercises below

1. Plot the slope field.
2. Graph the particular solution given.
3. Graph another particular solution over the specified interval (you are free to choose any particular solution that you like).
4. Use maple commands to find the general solutions of the ODE's, and particular solutions if possible.

1. A logistic equation

$$y' = y(2 - y), \quad y(0) = 1/2; \quad 0 \leq x \leq 4, \quad 0 \leq y \leq 3$$

2.

$$y' = (\sin x)(\sin y), \quad y(0) = 2; \quad -6 \leq x \leq 6, \quad -6 \leq y \leq 6$$