Individual Project 2

Model of Fishing in Plauer Lake

A mathematical model of trout fishing in Plauer Lake is considered.

The differential equation

$$dx/dt = (1 - x)x - c \qquad (1)$$

describes utilization of the fish population, where x(t) is the proportion of the maximum possible amount of fish in the lake (when there was no fishing) and t is time. The constant parameter ***c>0*** is the proportion of the fish population allowed for fishing. It characterises the allowed rate of the fishing-ground (intensity) and is called a **quota.** Choosing the values of the parameter ***c*** is an important factor of the control of the fish population.

Questions:

1) Solve equation (1). Separating the variables and completing the perfect square of the quadratic you will get 3 different solutions for different values of parameter c:

   a) $0 < c < \frac{1}{4}$ b) $c = \frac{1}{4}$ c) $c > \frac{1}{4}$.

   You can leave the solutions in an implicit form, that is not making x(t) the subject.

2) Find the number of equilibrium solutions equation (1) depending on the value of parameter c, that is solve the equation:

$$dx/dt = 0 \quad \text{or} \quad -x^2 + x - c = 0.$$

3) Draw direction fields and integral curves using Omnigraph or Matlab or Mathematica or Maple or any other program for the following values of parameter c: $a)\ c = \frac{1}{6}$ $b) c = \frac{1}{4}$ $c) c = \frac{1}{3}$

In each case a), b) and c) draw 10 integral curves for the following values of the initial condition x(0): 0.1;0.2;0.3;0.4;0.5;0.6;0.7;0.8;0.9;1.

State whether each equilibrium is ***stable*** or ***unstable***.

4) Interpret the equilibrium solutions by giving recommendation about permitted quotas for fishing.