Crystallography Reports, Vol. 41, No. 5, 1996, pp. 749–755. Translated from Kristallografiya, Vol. 41, No. 5, 1996, pp. 791–797.

Magnetooptical Interaction of Light with a Periodic Bigyrotropic Medium

O. V. Ivanov and D. I. Sementsov

Moscow State University, Ul'yanovsk Division, ul. L'va Tolstogo 42, Ul'yanovsk, 432700 Russia

Received October 26, 1995

Abstract—The specific features of the interaction of light with a periodic bigyrotropic medium are discussed. The dispersion relation is found for circularly polarized eigenwaves propagating along the periodicity axis of the medium. The reflection and transmission coefficients, the ellipticity, and the rotation angle of the polazization plane are found for light waves corresponding to the resonant and nonresonant frequency bands.

INTRODUCTION

Growth technologies currently available for thin-film single crystal structures with periodic distributions of the refractive index provide an opportunity to form one-dimensional structures with periods comparable to the wavelength of the electromagnetic radiation propagating in them. Such structures can be formed by employing various materials, such as semiconductors, magnetic materials, and liquid crystals. The advent of new magnetic materials transparent to infrared radiation and allowing for the fabrication of easily controlled domain and noncollinear magnetic structures has stimulated progress in magnetooptic studies of materials with periodic magnetic structures [1–4]. Of special interest are the media where the propagation of electromagnetic waves should be described taking into consideration both the electric and magnetic gyrotropies determined by the dielectric permittivity $\hat{\varepsilon}$ and magnetic permeability $\hat{\mu}$ tensors [5]. A typical example of such materials is the single crystal yttrium–iron garnet ($Y_3Fe_5O_{12}$) and its various modifications with partial substitution of rare-earth ions Bi, Lu, and Tb for yttrium ions [5, 6].

Up to now, the propagation of electromagnetic waves in bigyrotropic media was studied for systems with uniform magnetization [7–9] and for layered structures [10, 11]. In this paper, we discuss the specific features of electromagnetic wave propagation along the periodicity axis in a bigyrotropic medium characterized by a nonuniform magnetization with a continuous periodic distribution. As an example of such magnetic structure, we choose the structure of a longitudinal static spin wave. In this wave, the spin component along the periodicity axis varies harmonically, whereas the transverse spin components are either randomly oriented or are equal to zero [12, 13].

EQUATIONS FOR CIRCULAR WAVES

As the coordinate axes, let us choose three fourfold axes of a cubic crystal, and let the periodicity axis of the magnetic structure coincide with the [001] direction. Neglecting the quadratic magnetooptical coupling, we can write the dielectric permittivity and magnetic permeability tensors in the following form [5, 14]

$$\hat{\varepsilon} = \varepsilon\begin{pmatrix} 1 & if(z) & 0 \\ -if(z) & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \hat{\mu} = \mu\begin{pmatrix} 1 & ig(z) & 0 \\ -ig(z) & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{1}$$

The structure type under discussion can be characterized by the following spatial distribution of the magnetooptical parameters

$$f(z) = f_0 + f_1\cos qz, \quad g(z) = g_0 + g_1\cos qz, \tag{2}$$

where $q = 2\pi/D$ is the absolute value of the reciprocal lattice vector, D is the period of the magnetic structure, f_0 and g_0 are parameters related to the constant component of the magnetization, and f_1 and g_1 are parameters determined by the depth of magnetization modulation in the structure.

Light propagation along the periodicity axis in the crystal is described by the following equations determining the field components with the circular polarization $E_\pm = E_x \pm iE_y$ and $H_\pm = H_x \pm iH_y$,

$$\begin{aligned} E''_\pm \mp \frac{g'}{1+g}E'_\pm + k_0^2\varepsilon\mu(1\pm f)(1\pm g)E_\pm &= 0, \\ H''_\pm \mp \frac{f'}{1+f}H'_\pm + k_0^2\varepsilon\mu(1\pm f)(1\pm g)H_\pm &= 0, \end{aligned} \tag{3}$$

1063-7745/96/4105-0749$10.00 © 1996 МАИК Наука / Interperiodica Publishing

where the prime designates the derivatives with respect to the z coordinate, $k_0 = \omega/c$, ω is the frequency, and c is the speed of light in a vacuum. Let us introduce the following notation in (3): $f_1^{\pm} = \pm f_1$, $g_1^{\pm} = \pm g_1$ and

$$\kappa_{\pm} = k_0\sqrt{\varepsilon\mu(1 \pm (f_0 + g_0))},\ \ \gamma_{\pm} = \pm\frac{f_1 + g_1}{1 \pm (f_0 + g_0)}.$$

Then, bearing in mind that the magnetooptical parameters are small (for iron garnet in the bigyrotropic state, we have f and $g \sim 10^{-4}$–10^{-5}), we can rewrite set (3) in the form

$$\begin{aligned} &E''_{\pm} + qg_1^{\pm}\sin qzE'_{\pm} + \kappa_{\pm}^2(1 + \gamma_{\pm}\cos qz)E_{\pm} = 0, \\ &H''_{\pm} + qf_1^{\pm}\sin qzH'_{\pm} + \kappa_{\pm}^2(1 + \gamma_{\pm}\cos qz)H_{\pm} = 0. \end{aligned} \tag{4}$$

From these equations it follows that, in the case under discussion as well as in a medium with a uniform magnetization, the waves with circular polarization are the eigenwaves. This is related to the light propagation along the direction of magnetization. In a periodic medium, the bigyrotropy results in the appearance of additional terms in the wave equations, including the first derivative for the electric and magnetic fields. Similar equations appear also in the case of a monogyrotropic periodic medium, but they are valid only for one of the fields: either for the electric field in the case of magnetogyrotropy or for the magnetic field in the case of electrogyrotropy. As a rule, the analysis of wave propagation in such media is based on simpler wave equations involving either no term with E' or no term with H' [13].

NONRESONANT LIGHT–MEDIUM INTERACTION

The wave equations have the same form for both circular field components. Therefore, we omit below the sign superscripts "±" where they are not obligatory. Each equation in (4) can be presented in the form

$$G'' + qp_1\sin qz\, G' + \kappa^2(1 + \gamma\cos qz)G = 0, \tag{5}$$

where $p_1 = g_1$ for $G = E$ and $p_1 = f_1$ for $G = H$. The small parameters p_1 and γ have the same order of magnitude. This allows us to construct an approximate solution to equation (5) in the form $G = G_0 + G_1 + \ldots$, where G_i is proportional to the ith power of the above-mentioned small parameters. Substituting this solution into (5), we find an equation in G_0 and its solution

$$G''_0 + \kappa^2 G_0 = 0,\ \ G_0 = A_1e^{-i\kappa z} + A_2e^{i\kappa z}. \tag{6}$$

Substitution of (6) into (5) results in a correction of the next order,

$$\begin{aligned} G_1 = {}& \frac{\kappa(\kappa\gamma + p_1q)}{2q(q + 2\kappa)}(A_1e^{-i(\kappa+q)z} + A_2e^{i(\kappa+q)z}) \\ &+ \frac{\kappa(\kappa\gamma - p_1q)}{2q(q - 2\kappa)}(A_1e^{-i(\kappa-q)z} + A_2e^{i(\kappa-q)z}). \end{aligned} \tag{7}$$

Solution (7) is valid far from the values $\kappa = \pm q/2$ (the resonant spatial frequencies). In the vicinity of these values, the correction G_1 related to the periodic nonuniformity strongly perturbs the "uniform" solution (6). Taking into account the smallness of the gyrotropic parameters, we find the expressions for the ellipticity χ and for the rotation angle of the polarization plane θ in the case of a linearly polarized wave propagating in the medium

$$\begin{aligned} \chi &= \frac{f_1 - g_1}{4 - (q/\kappa)^2}(1 - \cos qz), \\ \theta &= \theta_0 z + \frac{\kappa}{q}\frac{2(f_1 + g_1) - g_1(q/\kappa)^2}{4 - (q/\kappa)^2}\sin qz, \end{aligned} \tag{8}$$

where $\theta_0 = \frac{\kappa}{2}(f_0 + g_0)$ is the specific rotation angles of the polarization plane in a uniformly magnetized sample, and $\kappa = k_0\sqrt{\varepsilon\mu}$. It follows from (8) that the wave retains its linear polarization at the points $z_1 = mD$, whereas the maximum deviation from the linear polarization occurs at the points $z_2 = (m + \frac{1}{2})D$. The rotation of the large axis of the polarization ellipse is characterized by oscillations superimposed on the linear dependence. At the points $mD/2$, the rotation of the ellipse is determined by only the constant component of magnetization. As the resonant frequency range is approached ($q \simeq \pm 2\kappa$), the light–medium interaction is enhanced. This manifests itself in large deviations from the linear polarization and in an increase in the amplitude of the oscillations of the rotation angle of the wave polarization ellipse.

The account taken of the following corrections G_n in (5) gives rise to the harmonics with higher spatial frequencies $\kappa \pm nq$. Their amplitudes are proportional to the nth power of the chosen small parameters, and, therefore, their contribution to the general solution can be ignored.

RESONANT LIGHT–MEDIUM INTERACTION

Let us discuss the specific features of the solution to equation (5) within the resonant frequency bands. We seek the solution in the vicinity of one of the main resonances $\kappa \simeq q/2$ as a superposition of two waves propagating in the opposite directions

$$G(z) = A(z)e^{-i\kappa z} + B(z)e^{i\kappa z}. \tag{9}$$

Assuming that the amplitudes $A(z)$ and $B(z)$ of the opposite waves are slowly changing functions, we can omit the second derivatives and the terms of the second order in the small parameters after substituting (9) into (5). Introducing the small deviation $\Delta = \kappa - q/2$ from the spatial resonance frequency, we find two equations for the coupled waves following from (5)

$$A' = -i\eta B e^{2i\Delta z}, \quad B' = i\eta A e^{-2i\Delta z}, \tag{10}$$

where $\eta = \kappa(f_1 - g_1)/4$ is the coupling constant for the direct and reverse waves. Thus, the interaction of opposite waves in the resonant frequency range is determined by the difference in the parameters characterizing the electro- and magnetogyrotropy rather than by their sum, as it is the case for the Faraday rotation. Media with f and g of the same sign exhibit pronounced gyrotropic characteristics related to the Faraday rotation, while the gyrotropic features of the resonant interaction are less distinct. Conversely, the resonant interaction is enhanced in the periodic bigyrotropic media, in which the gyrotropic parameters f and g are of opposite sign.

Solving equations (10), we find expressions for the amplitudes of the direct and reverse waves

$$\begin{aligned} A &= e^{i\Delta z}\{C_1 e^{-\nu z} + C_2 e^{\nu z}\}, \\ B &= -\frac{1}{\eta} e^{-i\Delta z}\{(\Delta + i\nu)C_1 e^{-\nu z} + (\Delta - i\nu)C_2 e^{\nu z}\}, \end{aligned} \tag{11}$$

where $\nu = \sqrt{\eta^2 - \Delta^2}$ and the coefficients $C_{1,2}$ are determined from the boundary conditions. The derived solutions are valid both within the resonant region (where $|\eta| > |\Delta|$ and ν is a real quantity) and outside this region (where $|\eta| < |\Delta|$ and ν is a purely imaginary quantity). System (10) has an integral corresponding to the total energy conservation for coupled waves propagating in opposite directions. Thus, we have for the resonant region

$$|A|^2 - |B|^2 = \frac{2\nu}{\nu + i\Delta} C_1 C_2^* + \mathrm{CC}, \tag{12}$$

where CC are the complex conjugate terms. For a semi-infinite medium ($z > 0$), the amplitudes of the two waves propagating in opposite directions are exponentially decreasing functions. For a wave with an amplitude $A(0)$ incident on the medium, the coefficients are $C_1 = A(0)$ and $C_2 = 0$. The resonant interaction of a light wave with the periodic bigyrotropic structure manifests itself in a strong attenuation of the direct wave because of the energy transfer to the reverse wave. The direct wave dies out and the amplitude of the reverse wave is increasing from zero (at $z \longrightarrow \infty$) to its maximum value upon exiting from the medium at $z = 0$

$$\begin{aligned} A(z) &= A(0) e^{(i\Delta - \nu)z}, \\ B(z) &= -\sqrt{\frac{i\Delta - \nu}{i\Delta + \nu}} A(0) e^{-(i\Delta + \nu)z}. \end{aligned} \tag{13}$$

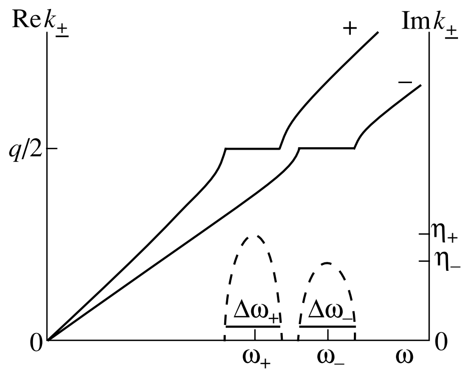

Fig. 1. Frequency dependences of the real parts (solid line) and imaginary parts (dashed line) of the propagation constants for circularly polarized waves.

It follows from (12) that the amplitudes of the direct and reverse waves of the semi-infinite medium meet the condition $|A(z)| = |B(z)|$. Therefore, for the above-mentioned range of parameter values, the resonant interaction of a light wave with the medium manifests itself in the effective Bragg reflection from the medium structure. Outside the resonant region (where ν is an imaginary quantity), the amplitudes are characterized by an oscillatory behavior.

From the solutions to equations (10) for the coupled waves, we get the following propagation constant for the circular waves

$$k = q/2 \pm i\sqrt{\eta^2 - \Delta^2}, \tag{14}$$

where the sign before the square root corresponds not to the type of circular polarization but to the choice of the corresponding branch of the dispersion curve at the point $k = q/2$. The frequency dependences of the real (solid line) and imaginary (dashed line) parts of the propagation constant are shown in Fig. 1. The dispersion curves $k_+(\omega)$ and $k_-(\omega)$ for the waves with right-hand and left-hand circular polarizations, respectively, undergo a discontinuity at $k_\pm = q/2$. As a result, the band gaps of width $\Delta\omega_\pm$ centered at the frequencies $\omega_\pm$ arise at these points,

$$\begin{aligned} \omega_\pm &= \frac{cq}{2\sqrt{\varepsilon\mu}}[1 \pm (f_0 + g_0)]^{-1/2}, \\ \Delta\omega_\pm &= \frac{1}{2}(f_1 - g_1)\omega_\pm. \end{aligned} \tag{15}$$

Within these gaps, we have a nonzero imaginary part of the propagation constant. For the corresponding polarizations, the waves with frequencies falling within the frequency bands $\Delta\omega_\pm$ decay rapidly. The waves with the right-hand polarization decay within the frequency band $\Delta\omega_+$, and the waves with the left-hand polarization decay within the frequency band $\Delta\omega_-$. It follows from (15) that, for a zero average magnetization ($f_0 = g_0 = 0$) or if the electric and magnetic gyrotropies compensate each other ($f_0 = -g_0$), the position and width of the band

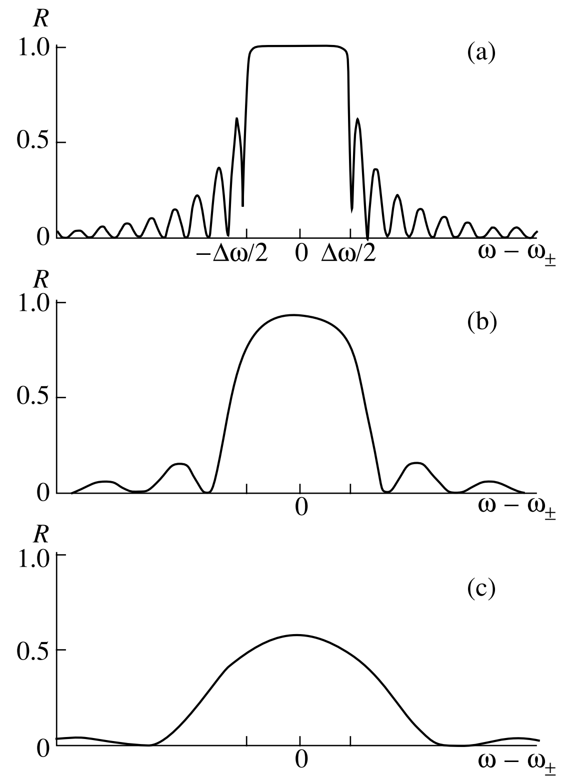

Fig. 2. Reflectivity versus frequency in the case of coincident resonant frequency bands for circularly polarized eigenwaves ($f_0 + g_0 = 0$) at different values of the normalized layer thickness: $L\eta$ = (a) 6, (b) 2, and (c) 1.

gaps is independent of the type of circular polarization of the waves interacting with the structure ($\omega_+ = \omega_-$ and $\Delta\omega_+ = \Delta\omega_-$). The width of the resonant frequency band is directly proportional to the difference between the parameters characterizing the modulation of the electro- and magnetogyrotropy.

TRANSMISSION AND REFLECTION OF LIGHT

The reflection and transmission of arbitrarily polarized radiation can be described by the following matrix relations

$$\begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = \begin{bmatrix} r_{xx} & r_{xy} \\ r_{yx} & r_{yy} \end{bmatrix} \begin{bmatrix} E_x^i \\ E_y^i \end{bmatrix}, \qquad (16)$$
$$\begin{bmatrix} E_x^t \\ E_y^t \end{bmatrix} = \begin{bmatrix} t_{xx} & t_{xy} \\ t_{yx} & t_{yy} \end{bmatrix} \begin{bmatrix} E_x^i \\ E_y^i \end{bmatrix},$$

where the superscripts i, r, and t correspond to the incident, reflected, and transmitted waves, respectively. The matrix elements can be found from the reflection and transmission coefficients for the waves with circular polarization

$$\begin{aligned} r_{xx} &= r_{yy} = (r_+ + r_-)/2, \\ r_{yx} &= -r_{xy} = (r_+ - r_-)/2i, \\ t_{xx} &= t_{yy} = (t_+ + t_-)/2, \\ t_{yx} &= -t_{xy} = (t_+ - t_-)/2i. \end{aligned} \qquad (17)$$

Using the above relations, we can find the reflectivity R (in terms of intensity), the rotation angle θ of the polarization plane, and the ellipticity χ [15]

$$\begin{aligned} R &= |r_{xx}|^2 + |r_{yx}|^2, \\ \tan 2\theta &= \tan 2\alpha \cos(\varphi - \psi), \\ \sin 2\chi &= \sin 2\alpha \sin(\varphi - \psi), \end{aligned} \qquad (18)$$

where $\tan\alpha = |r_{yx}|/|r_{xx}|$, $\psi = \arg(r_{xx})$, and $\varphi = \arg(r_{yx})$.

Let us discuss the specific features of the resonant interaction between a light wave and a bigyrotropic layer of thickness L placed in a uniform isotropic medium with the material constants ε and μ. Considering such a system as different media, we can write the wave fields in them as

$$\begin{aligned} E &= A_i e^{-i\kappa z} + B_i e^{i\kappa z}, \\ H &\simeq \pm i\sqrt{\varepsilon/\mu}(A_i e^{-i\kappa z} - B_i e^{i\kappa z}), \end{aligned} \qquad (19)$$

where i = 1, 2, 3 is the medium number. In the bigyrotropic layer (i = 2), the amplitudes A_i and B_i are determined by relations (11), and, outside the layer, they are constant. By solving the boundary problem, we arrive at the following expressions for the constants C_1 and C_2 in (11)

$$\begin{aligned} C_1 &= \frac{1}{2}\frac{(\nu + i\Delta)e^{\nu L}}{\nu\cosh\nu L + i\Delta\sinh\nu L}, \\ C_2 &= \frac{1}{2}\frac{(\nu - i\Delta)e^{-\nu L}}{\nu\cosh\nu L + i\Delta\sinh\nu L}, \end{aligned} \qquad (20)$$

and for the reflection and transmission coefficients characterizing the amplitudes of the circularly polarized waves,

$$\begin{aligned} r_\pm &= \frac{-i\eta_\pm \sinh(\nu_\pm L)}{\nu_\pm\cosh(\nu_\pm L) + i\Delta_\pm\sinh(\nu_\pm L)}, \\ t_\pm &= \frac{\nu_\pm e^{-iqL/2}}{\nu_\pm\cosh(\nu_\pm L) + i\Delta_\pm\sinh(\nu_\pm L)}. \end{aligned} \qquad (21)$$

In the special case that the electric and magnetic gyrotropies compensate each other ($f_0 + g_0 = 0$), we have $r_+ = -r_-$ and $t_+ = t_-$, hence

$$\begin{aligned} r_{xx} &= r_{yy} = 0, \quad t_{xy} = t_{yx} = 0, \\ r_{xy} &= -r_{yx} = ir_+, \quad t_{xx} = t_{yy} = t_+. \end{aligned}$$

Therefore, if the incident light is linearly polarized, then the reflected light is also linearly polarized with the polarization plane rotated through 90°. In this case, the transmitted light does not change its polarization.

In the case under discussion, the reflection and transmission coefficients (in terms of intensity) are determined as follows

$$R = \frac{\eta^2 \sinh^2 \nu L}{\eta^2 \sinh^2 \nu L + \nu^2}, \quad T = \frac{\nu^2}{\eta^2 \sinh^2 \nu L + \nu^2}. \qquad (22)$$

For the samples with a large number of magnetization oscillations ($\eta L > 1$), the reflectivity is close to unity within the resonant interaction range, whereas the transmission coefficient is close to zero. This is the result of an intense Bragg reflection of light from the periodic structure (Fig. 2). Outside the resonant region, the coefficients R and T are found by substituting $i\nu$ for ν in (22)

$$R = \frac{\eta^2 \sin^2 |\nu| L}{\eta^2 \sin^2 |\nu| L + |\nu|^2}, \quad T = \frac{|\nu|^2}{\eta^2 \sin^2 |\nu| L + |\nu|^2}, \qquad (23)$$

and their dependences on the thickness are purely oscillatory. These oscillations stem from the resonant mechanism of light interaction with the structure. For samples with a small number of oscillation periods ($\eta L < 1$), a significant part of the light wave energy is transmitted through the sample, even though the frequency falls within the resonant range.

Using relations (17) for the matrix elements and relations (21) for the reflectivity, we find the phase shift for the reflected light

$$\varphi = \arctan\left(\frac{\Delta}{\nu} \tanh \nu L\right). \qquad (24)$$

The plots of the phase shift for the reflected light versus the layer thickness normalized to the coupling constant ηL are presented in Fig. 3 for different deviations (detuning) Δ from the center of coincident resonant frequency bands. Curve *3* corresponds to the boundary of the resonant frequency band ($\nu = 0$). For the layer thickness tending to infinity, the phase shift is equal to $\pi/2$. Outside the resonant band, we have

$$\varphi = \arctan\left(\frac{\Delta}{|\nu|} \tan |\nu| L\right), \qquad (25)$$

and the monotonic behavior of the curve changes to oscillatory behavior. The period of the oscillations decreases with the increase in the detuning (off the resonance) (curves *4*–*6*).

If the structure magnetization has a constant component, the resonant frequency ranges do not coincide for the right-hand and left-hand circularly polarized waves ($\kappa_+ = q/2$ for the right-hand circular polarization and $\kappa_- = q/2$ for the left-hand circular polarization). The corresponding Δ and ν parameters differ for different polarizations. As a result, all terms in the reflection and transmission matrices are nonzero, and the light acquires an ellipticity, a rotation of the polarization plane, and a phase shift. Two mutual arrangements of the resonant frequency bands are possible: with an overlap, which occurs if the average value of the magnetization is sufficiently small $f_0 + g_0 < (f_1 - g_1)/2$

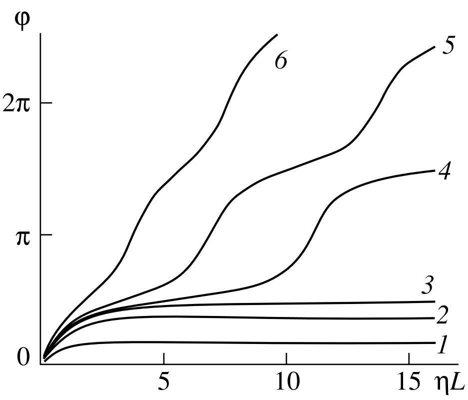

Fig. 3. Phase shift of the reflected wave versus normalized thickness of the bigyrotropic layer in the case $f_0 + g_0 = 0$ at different deviations (detuning) from the center of the resonant frequency band: Δ/η = (*1*) 0.5, (*1*) 0.9, (*3*) 1, (*4*) 1.04, (*5*) 1.1, and (*6*) 1.3.

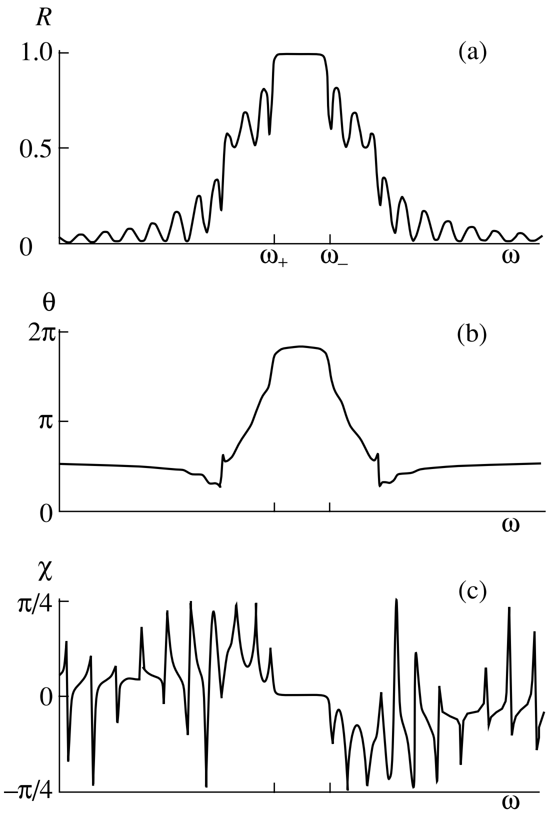

Fig. 4. (a) Reflectivity, (b) rotation angle of the polarization plane, and (c) ellipticity versus frequency in the case of partially overlapping ($f_0 + g_0 = (f_1 - g_1)/4$) resonant frequency of circularly polarized waves; $L\eta = 6$.

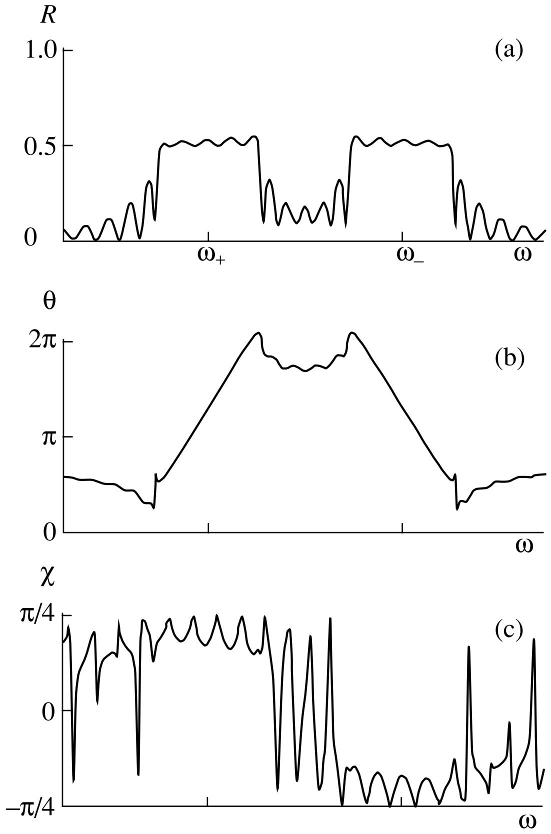

Fig. 5. (a) Reflectivity, (b) rotation angle of the polarization plane rotation, and (c) ellipticity versus frequency in the case of nonoverlaping ($f_0 + g_0 = f_1 - g_1$) resonant frequency of circularly polarized waves; $L\eta = 6$.

(Fig. 4), and with no overlap if $f_0 + g_0 > (f_1 - g_1)/2$ (Fig. 5). Within the overlap of the band gaps, the reflectivity is close to unity, and, outside it, the oscillations corresponding to the "tail" of one band are superimposed on the reflectivity (equal to 50%) of another band (Figs. 4a and 5a). In the resonant frequency band, the rotation angle of the polarization plane is linearly increasing for the right-hand circular polarization and decreasing for the left-hand circular polarization (Figs. 4b and 5b). As we might expect, in the resonant frequency band for the right-hand circular polarization, the reflection is related mainly to the wave of the same polarization, while the wave with the left-hand circular polarization propagates without interaction. Thus, the resulting ellipticity is $\chi \approx \pi/4$. In the resonant frequency band for the left-hand circular polarization, the situation is reversed, and we have $\chi \approx -\pi/4$ (Figs. 4c and 5c).

Let us discuss the reflection of a light wave incident onto an infinite periodic bigyrotropic medium from the side of a uniform isotropic dielectric medium characterized by the permittivity ε_1 and permeability μ_1. In this case, we have a superposition of two effects: first, the reflection from the medium as from a dielectric and second, the reflection from a bigyrotropic periodic structure as a result of the resonant interaction. Solving the boundary problem for the electromagnetic field, we find the reflection coefficients

$$r_{\pm} = -\frac{\eta_{\pm}\rho + (\Delta_{\pm} + i\nu_{\pm})}{\eta_{\pm} + \rho(\Delta_{\pm} + i\nu_{\pm})}, \tag{26}$$

where

$$\rho = \frac{\sqrt{\varepsilon/\mu} - \sqrt{\varepsilon_1/\mu_1}}{\sqrt{\varepsilon/\mu} + \sqrt{\varepsilon_1/\mu_1}}.$$

This expression is valid within the resonant frequency band but not outside it. Therefore, if we want to calculate the reflection coefficients for linearly polarized waves, we should consider the frequencies within the resonant frequency band for both the right-hand and the left-hand circular polarizations. If $f_0 + g_0 = 0$, the reflected light is linearly polarized with the polarization plane rotated through an angle θ

$$\tan\theta = \frac{\eta}{\nu}\frac{1-\rho^2}{2\rho}. \tag{27}$$

Hence, it follows that, if the dielectric constants of the adjacent media are equal, the polarization plane will be rotated by $\pi/2$ for all frequencies within the resonant frequency band. Similarly, at the boundaries of the resonant frequency band, the rotation angle of the polarization plane equals $\pi/2$ independent of the ratio between the dielectric constants. The absolute value of the rotation angle of the polarization plane is a minimum at the center of the resonant region. This minimum is deeper, the larger the difference is between the permittivities.

The above analysis demonstrates that, in bigyrotropic media with f and g of the same sign, the contributions of the two gyrotropies to the Faraday rotation are added, whereas, for the resonant interaction of light with a periodic structure, the contributions of the two gyrotropies are subtracted. Therefore, intense resonant interaction of light with a periodic magnetic structure in a bigyrotropic medium can be observed if f and g have opposite signs.

REFERENCES

1. Zvezdin, A.K. and Kotov, V.A., *Magnitooptika tonkikh plenok* (Magnetooptics of Thin Films), Moscow: Nauka, 1988.
2. Randoshkin, V.V. and Chervonenkis, A.Ya., *Prikladnaya magnitooptika* (Applied Magnetooptics), Moscow: Energoatomizdat, 1990.
3. Erukhimov, M.Sh. and Tyurnev, V.V., *Fiz. Tverd. Tela* (Leningrad), 1975, vol. 17, no. 8, p. 2440; 1976, vol.18, no. 6, p. 1635.
4. Sementsov, D.I., *Opt. Spektrosk.*, 1981, vol. 50, no. 1, p. 37.

5. Krinchik, G.S., *Fizika magnitnykh yavlenii* (Physics of Magnetic Phenomena), Moscow: Mosk. Gos. Univ., 1985.
6. Ageev, A.N., Belitskii, S.N., Kitaev, S.A., Trifonov, A.S., and Gridnev, V.N., *Zh. Eksp. Teor. Fiz.*, 1990, vol. 98, p. 1390.
7. Fedorov, F.N., *Teoriya girotropii* (Theory of Gyrotropy), Minsk: Nauka i Tekhnika, 1976.
8. Eritsyan, O.S., *Usp. Fiz. Nauk*, 1982, vol, 138, p. 645.
9. Borisov, S.B. and Lyubchanskii, I.L., *Opt. Spektrosk.*, 1988, vol. 65, no. 2, p. 365.
10. Damascos, N.J., Maffet, A.L., and Uslenghi, P.I.E., *J. Opt. Soc. Am. A: Opt. Image Sci.*, 1985, vol. 2, p. 454.
11. Borisov, S.B., Dadoenkova, N.N., and Lyubchanskii, I.L., *Opt. Spektrosk.*, 1993, vol. 74, no. 4, p. 1127.
12. Eremenko, V.V., *Vvedenie v opticheskuyu spektroskopiyu magnetikov* (Introduction to Optical Spectroscopy of Magnetic Materials), Kiev: Naukova Dumka, 1975.
13. Sementsov, D.I., *Fiz. Nizk. Temp.* (Kharkov), 1980, vol. 6, no. 7, p. 906.
14. Ivanov, O.V. and Sementsov, D.I., *Kristallografiya*, 1995, no. 1, p. 89.
15. Born, M. and Wolf, E., *Principles of Optics: Electromagnetic Theory of Propagation, Interference, and Diffraction of Light*, Oxford: Pergamon, 1964.

Translated by T. Galkina