

Computation with Absolutely No Space Overhead

Lane Hemaspaandra¹ Proshanto Mukherji¹ Till Tantau²

¹Department of Computer Science
University of Rochester

²Fakultät für Elektrotechnik und Informatik
Technical University of Berlin

Developments in Language Theory Conference, 2003

Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful

Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful

Outline

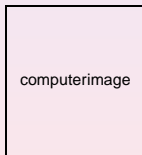
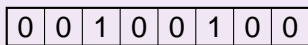
- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful

Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful

The Standard Model of Linear Space

tape



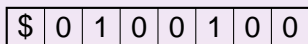
Turing machine

Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape



computerworkingimage

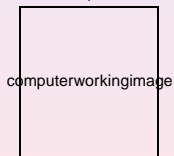
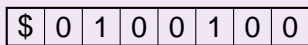
Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape



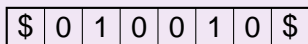
Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape



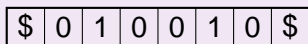
Turing machine

Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape



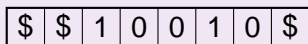
Turing machine

Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape



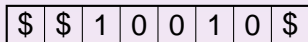
Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape



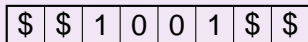
Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape



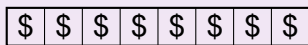
Turing machine

Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape



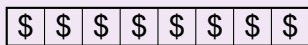
Turing machine

Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape

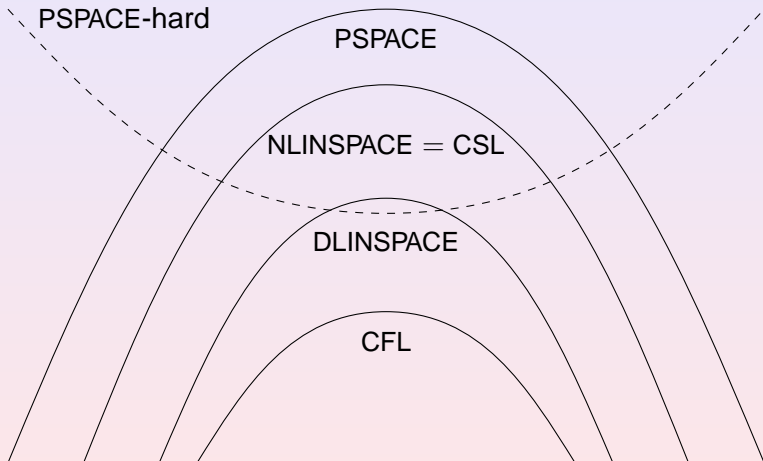


Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

Linear Space is a Powerful Model

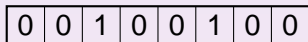


Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful

Our Model of “Absolutely No Space Overhead”

tape



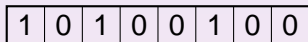
Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **equals** input alphabet

Our Model of “Absolutely No Space Overhead”

tape



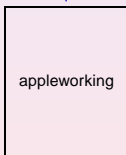
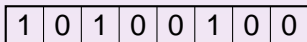
Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **equals** input alphabet

Our Model of “Absolutely No Space Overhead”

tape



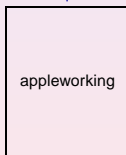
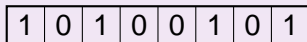
Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **equals** input alphabet

Our Model of “Absolutely No Space Overhead”

tape



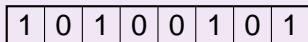
Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **equals** input alphabet

Our Model of “Absolutely No Space Overhead”

tape



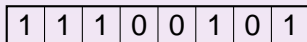
Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **equals** input alphabet

Our Model of “Absolutely No Space Overhead”

tape

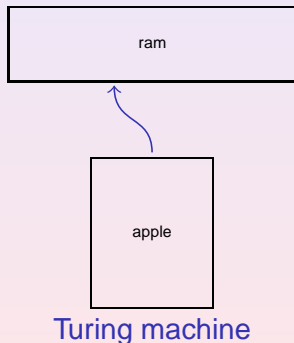


Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **equals** input alphabet

Our Model of “Absolutely No Space Overhead”



Intuition

- Tape is used like a RAM module.

Definition of Overhead-Free Computations

Definition

A Turing machine is **overhead-free** if

- 1 it has only a single tape,
- 2 writes only on input cells,
- 3 writes only symbols drawn from the input alphabet.

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ ,

DOF_{poly} if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.

NOF is the nondeterministic version of DOF,

NOF_{poly} is the nondeterministic version of DOF_{poly} .

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ ,

DOF_{poly} if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.

NOF is the nondeterministic version of DOF,

NOF_{poly} is the nondeterministic version of DOF_{poly}.

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ ,

DOF_{poly} if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.

NOF is the nondeterministic version of DOF,

NOF_{poly} is the nondeterministic version of DOF_{poly}.

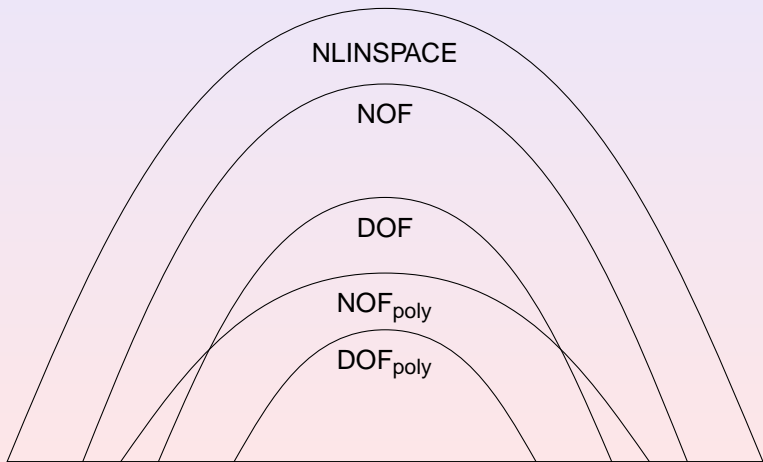
Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

- DOF** if L is accepted by a deterministic overhead-free machine with input alphabet Σ ,
- DOF_{poly}** if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.
- NOF** is the nondeterministic version of DOF,
- NOF_{poly}** is the nondeterministic version of DOF_{poly}.

Simple Relationships among Overhead-Free Computation Classes

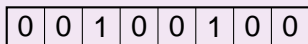


Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

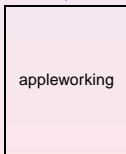
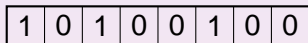
Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

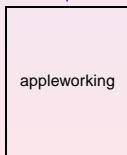
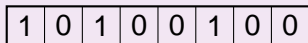
Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

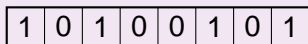
Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

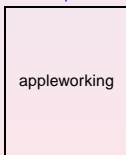
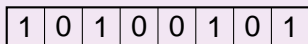
- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers**
- Find left end marker
- Advance left end marker
- Find right end marker
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

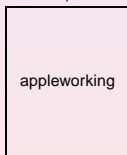
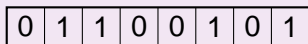
Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

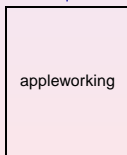
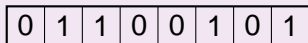
- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers
- Find left end marker
- Advance left end marker**
- Find right end marker
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

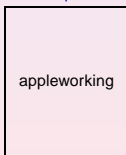
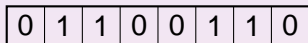
- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker**
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

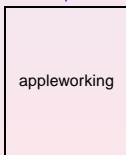
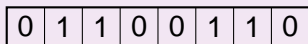
- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker
- Advance right end marker**

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

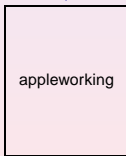
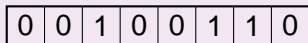
Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

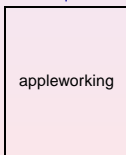
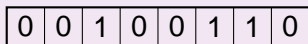
- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers
- Find left end marker
- Advance left end marker**
- Find right end marker
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

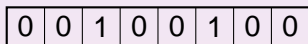
- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker**
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

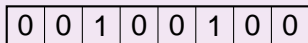
Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

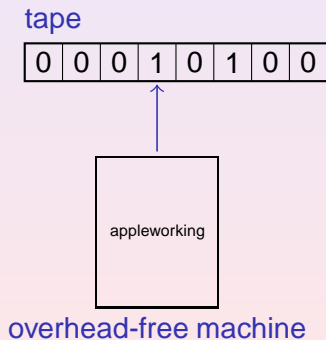
Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way



Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

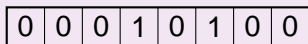
Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

Phase 1:

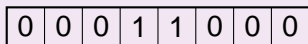
- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker**
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape



overhead-free machine

Algorithm

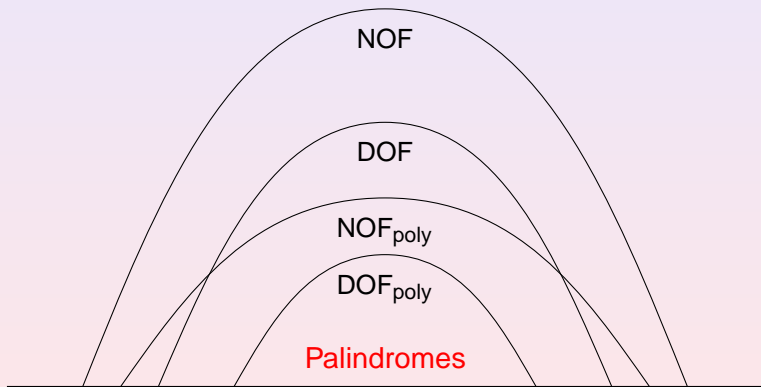
Phase 1:

- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker
- Advance right end marker**

Relationships among Overhead-Free Computation Classes



Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - **Linear Languages**
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful

A Review of Linear Grammars

Definition

A grammar is **linear** if it is context-free and there is only one nonterminal per right-hand side.

Example

$G_1: S \rightarrow 00S0 \mid 1$ and $G_2: S \rightarrow 0S10 \mid 0$.

Definition

A grammar is **deterministic** if
“there is always only one rule that can be applied.”

Example

$G_1: S \rightarrow 00S0 \mid 1$ is deterministic.

$G_2: S \rightarrow 0S10 \mid 0$ is **not** deterministic.

A Review of Linear Grammars

Definition

A grammar is **linear** if it is context-free and there is only one nonterminal per right-hand side.

Example

$G_1: S \rightarrow 00S0 \mid 1$ and $G_2: S \rightarrow 0S10 \mid 0$.

Definition

A grammar is **deterministic** if “there is always only one rule that can be applied.”

Example

$G_1: S \rightarrow 00S0 \mid 1$ is deterministic.

$G_2: S \rightarrow 0S10 \mid 0$ is **not** deterministic.

Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every deterministic linear language is in DOF_{poly} .

Metalinear Languages Can Be Accepted in an Overhead-Free Way

Definition

A language is **metalinear** if it is the concatenation of linear languages.

Example

TRIPLE-PALINDROME = $\{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Theorem

Every metalinear language is in NOF_{poly} .

Metalinear Languages Can Be Accepted in an Overhead-Free Way

Definition

A language is **metalinear** if it is the concatenation of linear languages.

Example

TRIPLE-PALINDROME = $\{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Theorem

Every metalinear language is in NOF_{poly} .

Metalinear Languages Can Be Accepted in an Overhead-Free Way

Definition

A language is **metalinear** if it is the concatenation of linear languages.

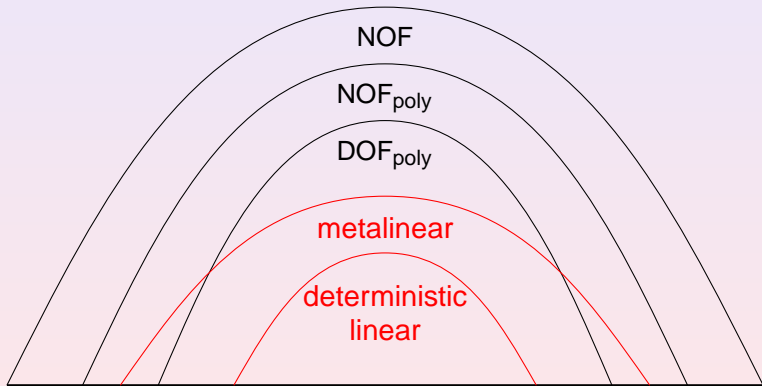
Example

TRIPLE-PALINDROME = $\{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Theorem

Every metalinear language is in NOF_{poly} .

Relationships among Overhead-Free Computation Classes



Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - **Context-Free Languages with a Forbidden Subword**
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful

Definition of Almost-Overhead-Free Computations

Definition

A Turing machine is **almost-overhead-free** if

- 1 it has only a single tape,
- 2 writes only on input cells,
- 3 writes only symbols drawn from the input alphabet plus one special symbol.

Definition of Almost-Overhead-Free Computations

Definition

A Turing machine is **almost-overhead-free** if

- 1 it has only a single tape,
- 2 **writes only on input cells,**
- 3 writes only symbols drawn from the input alphabet plus one special symbol.

Definition of Almost-Overhead-Free Computations

Definition

A Turing machine is **almost-overhead-free** if

- 1 it has only a single tape,
- 2 writes only on input cells,
- 3 **writes only symbols drawn from the input alphabet plus one special symbol.**

Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

*Let L be a context-free language with a forbidden word.
Then $L \in \text{NOF}_{\text{poly}}$.*

▶ Skip proof

Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

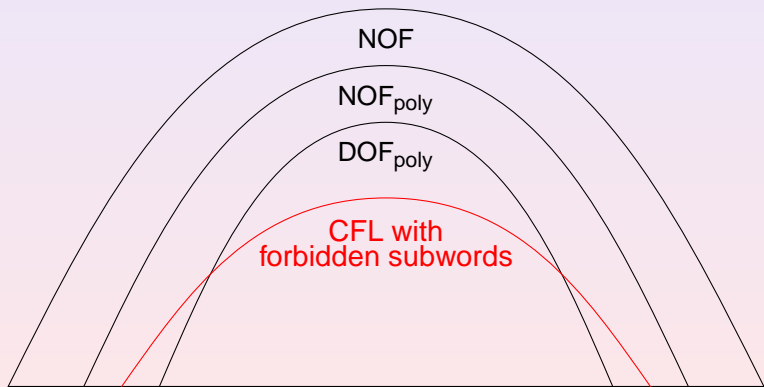
Theorem

*Let L be a context-free language with a forbidden word.
Then $L \in \text{NOF}_{\text{poly}}$.*

Proof.

Every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time. □

Relationships among Overhead-Free Computation Classes



Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful

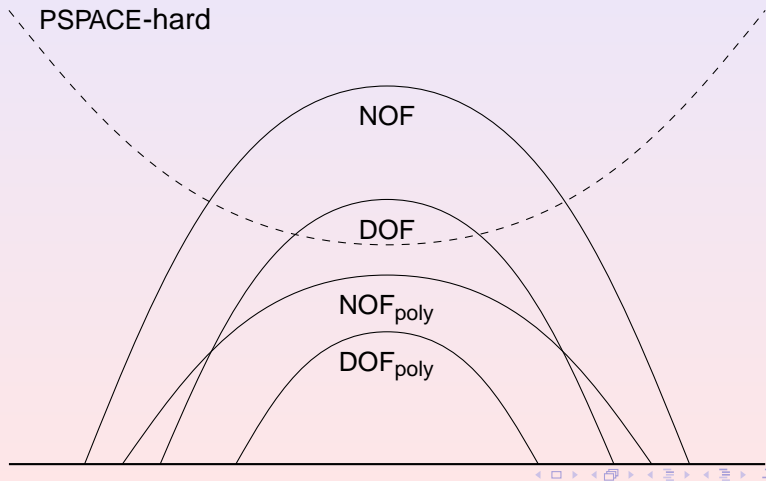
Overhead-Free Languages can be PSPACE-Complete

Theorem

DOF contains languages that are complete for PSPACE.

▶ Proof details

Relationships among Overhead-Free Computation Classes



Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful

Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

Theorem

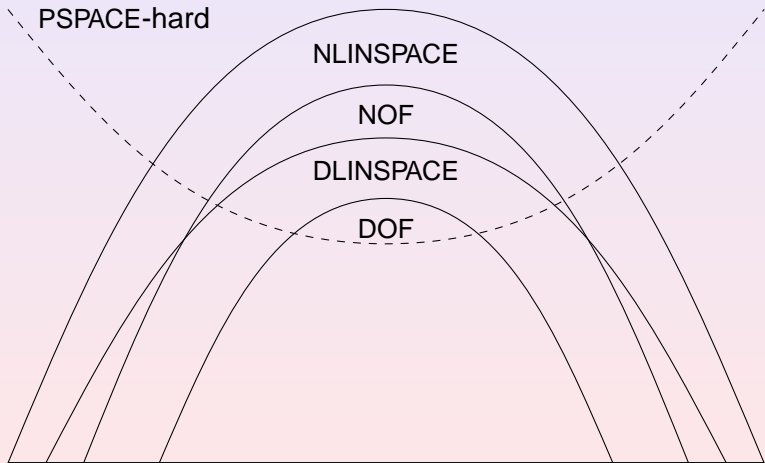
$\text{DOF} \subsetneq \text{DLINSPACE}$.

Theorem

$\text{NOF} \subsetneq \text{NLINSPACE}$.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

Relationships among Overhead-Free Computation Classes



Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Conjecture

DOUBLE-PALINDROMES \notin DOF.

Conjecture

$\{ww \mid w \in \{0, 1\}^*\} \notin$ NOF.

Proving the first conjecture would show $\text{DOF} \subsetneq \text{NOF}$.

Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Theorem

$\text{DOUBLE-PALINDROMES} \in \text{DOF}$.

Conjecture

$\{ww \mid w \in \{0, 1\}^*\} \notin \text{NOF}$.

Proving the first conjecture would show $\text{DOF} \subsetneq \text{NOF}$.

Summary

- Overhead-free computation is a more faithful **model of fixed-size memory**.
- Overhead-free computation is **less powerful** than linear space.
- **Many** context-free languages can be accepted by overhead-free machines.
- We conjecture that **all** context-free languages are in NOF_{poly} .
- Our results can be seen as new results on the power of **linear bounded automata with fixed alphabet size**.

For Further Reading



A. Salomaa.

Formal Languages.

Academic Press, 1973.



E. Dijkstra.

Smoothsort, an alternative for sorting in situ.

Science of Computer Programming, 1(3):223–233, 1982.



E. Feldman and J. Owings, Jr.

A class of universal linear bounded automata.

Information Sciences, 6:187–190, 1973.



P. Jančar, F. Mráz, M. Plátek, and J. Vogel.

Restarting automata.

FCT Conference 1995, LNCS 985, pages 282–292. 1995.

For Further Reading



A. Salomaa.

Formal Languages.

Academic Press, 1973.



E. Dijkstra.

Smoothsort, an alternative for sorting in situ.

Science of Computer Programming, 1(3):223–233, 1982.



E. Feldman and J. Owings, Jr.

A class of universal linear bounded automata.

Information Sciences, 6:187–190, 1973.



P. Jančar, F. Mráz, M. Plátek, and J. Vogel.

Restarting automata.

FCT Conference 1995, LNCS 985, pages 282–292. 1995.

For Further Reading



A. Salomaa.

Formal Languages.

Academic Press, 1973.



E. Dijkstra.

Smoothsort, an alternative for sorting in situ.

Science of Computer Programming, 1(3):223–233, 1982.



E. Feldman and J. Owings, Jr.

A class of universal linear bounded automata.

Information Sciences, 6:187–190, 1973.



P. Jančar, F. Mráz, M. Plátek, and J. Vogel.

Restarting automata.

FCT Conference 1995, LNCS 985, pages 282–292. 1995.

For Further Reading



A. Salomaa.

Formal Languages.

Academic Press, 1973.



E. Dijkstra.

Smoothsort, an alternative for sorting in situ.

Science of Computer Programming, 1(3):223–233, 1982.



E. Feldman and J. Owings, Jr.

A class of universal linear bounded automata.

Information Sciences, 6:187–190, 1973.



P. Jančar, F. Mráz, M. Plátek, and J. Vogel.

Restarting automata.

FCT Conference 1995, LNCS 985, pages 282–292. 1995.

Appendix Outline

- 4 Appendix
 - Complete Languages
 - Improvements for Context-Free Languages

tu-logo

ur-logo

Overhead-Free Languages can be PSPACE-Complete

Theorem

DOF contains languages that are complete for PSPACE.

Proof.

- 1 Let $A \in \text{DLINSPACE}$ be PSPACE-complete. Such languages are known to exist.
- 2 Let M be a linear space machine that accepts $A \subseteq \{0, 1\}^*$ with tape alphabet Γ .
- 3 Let $h: \Gamma \rightarrow \{0, 1\}^*$ be an isometric, injective homomorphism.
- 4 Then $h(L)$ is in DOF and it is PSPACE-complete. □

Improvements

Theorem

- 1 $\text{DCFL} \subseteq \text{DOF}_{\text{poly}}$.
- 2 $\text{CFL} \subseteq \text{NOF}_{\text{poly}}$.