Suggested Solution to Project 2

Model of Fishing in Plauer Lake

1) Equation (1) is a separable differential equation which has 3 different solutions depending on the value of parameter c.

Separating the variables and completing the perfect square we obtain:

$$\frac{dx}{x^2 - x + c} = -dt \quad \Rightarrow \quad \frac{dx}{(x-\frac{1}{2})^2 + (c-\frac{1}{4})} = -dt$$

After integrating both sides we obtain the following 3 different solutions (in an implicit form):

$$\frac{1}{2\sqrt{\frac{1}{4}-c}} \ln\left|\frac{x-\frac{1}{2}-\sqrt{\frac{1}{4}-c}}{x-\frac{1}{2}+\sqrt{\frac{1}{4}-c}}\right| = -t + C_1, \qquad if \;\; 0 < c < \frac{1}{4}$$

$$\frac{1}{x-\frac{1}{2}} = t + C_1, \qquad if \;\; c = \frac{1}{4}$$

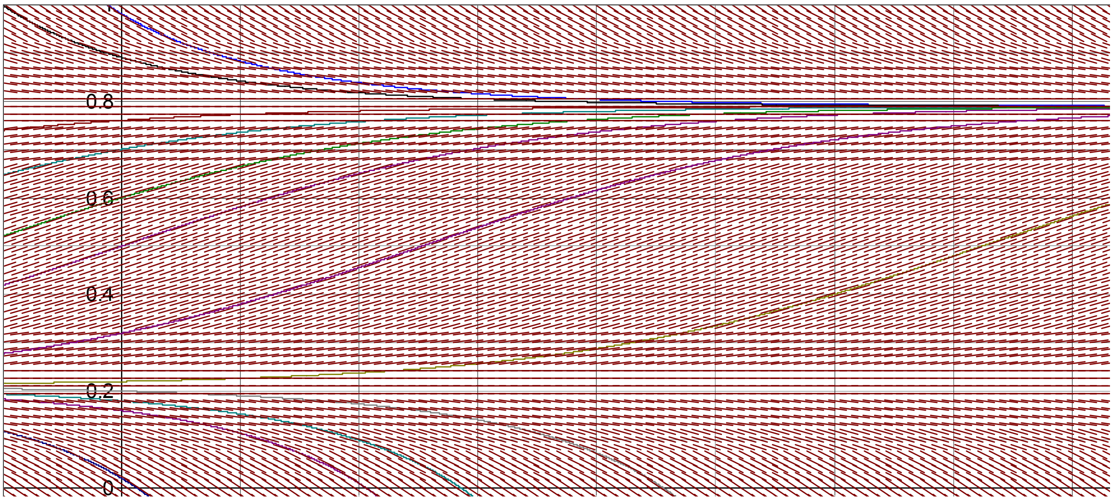
$$\frac{1}{\sqrt{c-\frac{1}{4}}} \tan^{-1}\frac{x-\frac{1}{2}}{\sqrt{c-\frac{1}{4}}} = -t + C_1, \qquad if \;\; c > \frac{1}{4}$$

2) The discriminant of the quadratic equation **$- x^2 + x - c = 0$** equals **D = 1 - 4c**. Depending on the value of the parameter ***c*** the discriminant can be greater than zero, equal to zero or less than zero which gives us two equilibrium solutions, one equilibrium solution or none:

1) D>0 : 0<c<1/4; 2 equilibrium solutions
2) D=0 : c=1/4; 1 equilibrium solution
3) D<0 : c>1/4; no equilibrium solutions

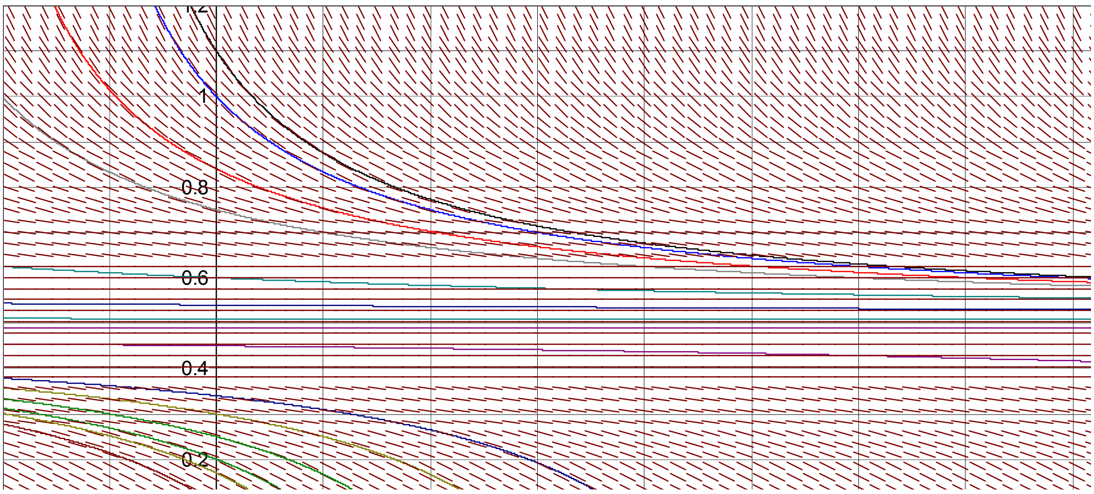
3) Drawing direction fields and integral curves using Omnigraph:

1. $0<c<1/4$ $\qquad$ $c=1/6$



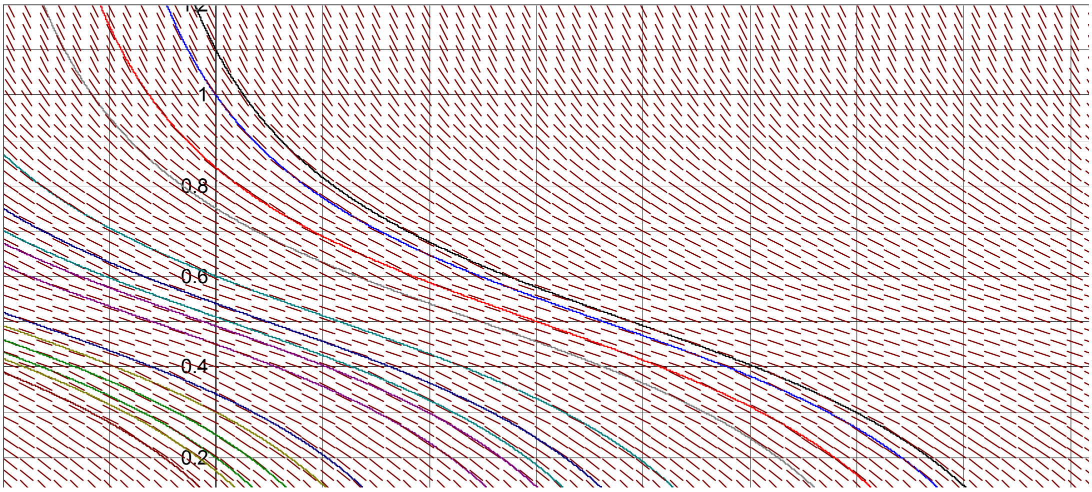
There are 2 equilibrium solutions. The upper equilibrium solution is stable (solutions near it tend to it as t increases) and the lower equilibrium solution is unstable (solutions near it move away from it as t increases).

2. $c=1/4$



There is one unstable equilibrium solution $x=0.5$. The quota $c=1/4$ is not allowed, since any small decrease of the proportion of the fish population from the equilibrium solution leads to the situation when the fish population would be completely fished out in a finite period of time.

3. $c>1/4$ $\qquad$ $c=1/3$



There are no equilibrium solutions. and the fish population will disappear during a finite period of time.

4) Theoretically any quota less than 1/4 is permitted up to the maximum c=1/4, but practically the quotas close to 1/4 are not allowed since even small decrease from the equilibrium solution (x=0.5) leads the fish population to become extinct.