Suggested Solution to Project 3

Model of Bird Population on Rügen Island

a) Separating the variables in the equation $\frac{dP}{dt} = 0.001(1200 - P)(P - 100)$ we obtain

$$\frac{dP}{(1200 - P)(P - 100)} = 0.001dt .$$

Using partial fraction decomposition on the left-hand side and multiplying both sides by 1100, we obtain

$$\left[\frac{1}{1200 - P} + \frac{1}{P - 100}\right]dP = 1.1dt$$

$$\ln|P - 100| - \ln|1200 - P| = 1.1t + C$$

$$\ln\left|\frac{P - 100}{1200 - P}\right| = 1.1t + C$$

$$\left|\frac{P - 100 \pm 1200}{1200 - P}\right| = \left|\frac{1100}{1200 - P} - 1\right|$$

$$\left|\frac{1100}{1200 - P} - 1\right| = e^C e^{1.1t}$$

$$\frac{1100}{1200 - P} = 1 + Ae^{1.1t}, \quad A = \pm e^C$$

$P = 1200 - \frac{1100}{Ae^{1.1t} + 1}$ is the general solution of the differential equation.

b) Using the initial condition $P(0) = 300$ we obtain

$$300 = 1200 - \frac{1100}{A + 1} \Rightarrow A = \frac{2}{9} .$$

Therefore

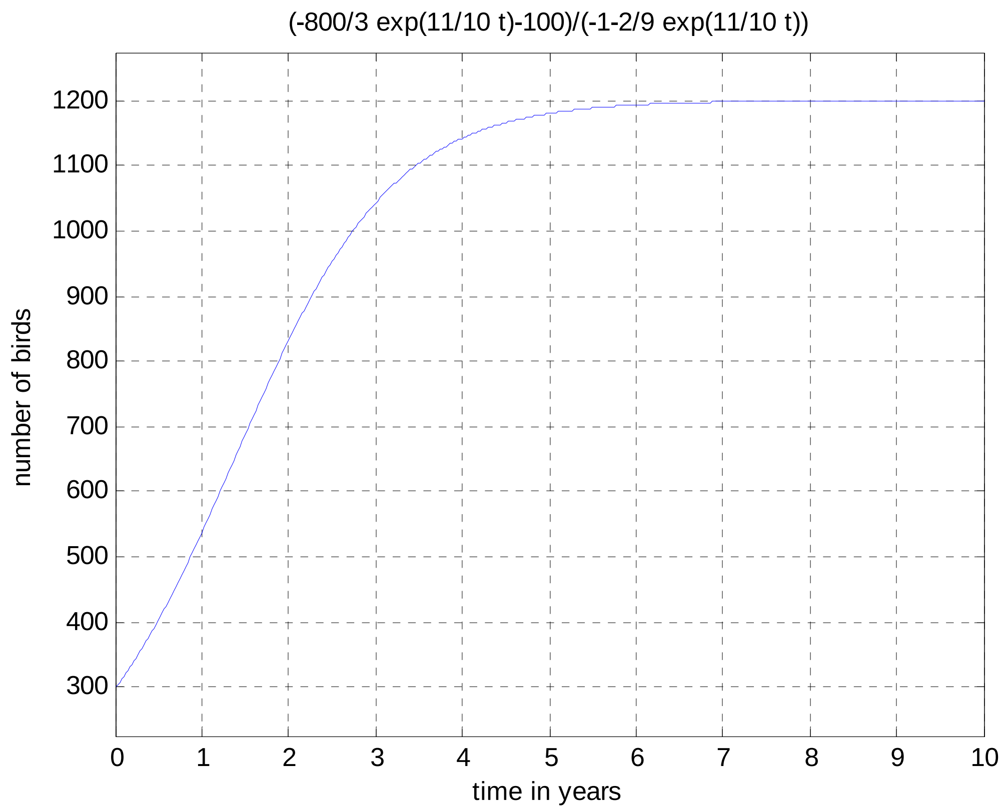
$P(t) = 1200 - \frac{1100}{\frac{2}{9}e^{1.1t} + 1}$. Substituting *t* = 5 we obtain $P(5) \approx 1180$.

c) P=dsolve('DP=0.001*(1200-P)*(P-100)','t')

P = (-800/3*exp(11/10*t)-100)/(-1-2/9*exp(11/10*t)) or

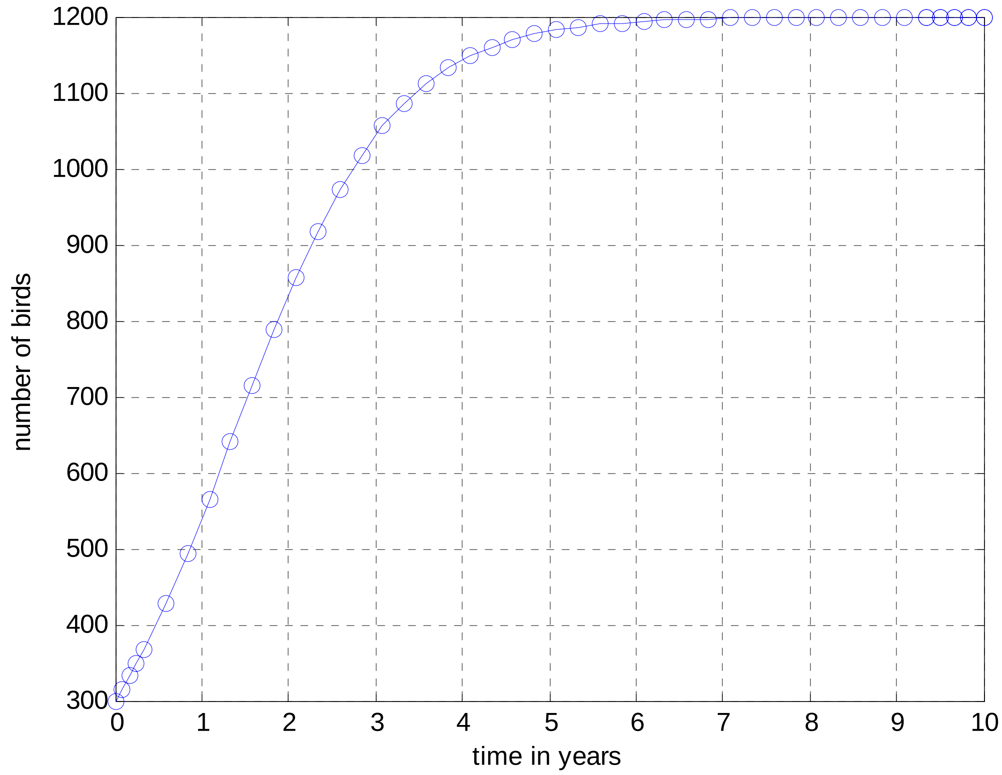
P = 300*(8*exp(11/10*t)+3)/(9+2*exp(11/10*t))

ezplot(P,[0,10])



g=inline('0.001*(1200-P)*(P-100)','t','P');

ode45(g,[0,10],300)



d) From the graph we can read that the curve is the steepest when $t = 1.5$, that is 1.5 years from now.