## Computer Programs by Chapter and Section

| 1.0 | flmoon | calculate phases of the moon by date |
| :--- | :--- | :--- |
| 1.1 | julday | Julian Day number from calendar date |
| 1.1 | badluk | Friday the 13th when the moon is full |
| 1.1 | caldat | calendar date from Julian day number |
|  |  |  |
| 2.1 | gaussj | Gauss-Jordan matrix inversion and linear equation |
|  |  | solution |
| 2.3 | ludcmp | linear equation solution, $L U$ decomposition |
| 2.3 | lubksb | linear equation solution, backsubstitution |
| 2.4 | tridag | solution of tridiagonal systems |
| 2.4 | banmul | multiply vector by band diagonal matrix |
| 2.4 | bandec | band diagonal systems, decomposition |
| 2.4 | banbks | band diagonal systems, backsubstitution |
| 2.5 | mprove | linear equation solution, iterative improvement |
| 2.6 | svbksb | singular value backsubstitution |
| 2.6 | svdcmp | singular value decomposition of a matrix |
| 2.6 | pythag | calculate $\left(a^{2}+b^{2}\right)^{1 / 2}$ without overflow |
| 2.7 | cyclic | solution of cyclic tridiagonal systems |
| 2.7 | sprsin | convert matrix to sparse format |
| 2.7 | sprsax | product of sparse matrix and vector |
| 2.7 | sprstx | product of transpose sparse matrix and vector |
| 2.7 | sprstp | transpose of sparse matrix |
| 2.7 | sprspm | pattern multiply two sparse matrices |
| 2.7 | sprstm | threshold multiply two sparse matrices |
| 2.7 | linbcg | biconjugate gradient solution of sparse systems |
| 2.7 | snrm | used by linbcg for vector norm |
| 2.7 | atimes | used by linbcg for sparse multiplication |
| 2.7 | asolve | used by linbcg for preconditioner |
| 2.8 | vander | solve Vandermonde systems |
| 2.8 | toeplz | solve Toeplitz systems |
| 2.9 | choldc | Cholesky decomposition |
| 2.9 | cholsl | Cholesky backsubstitution |
| 2.10 | qrdcmp | QR decomposition |
| 2.10 | qrsolv | QR backsubstitution |
| 2.10 | rsolv | right triangular backsubstitution |
| 2.10 | qrupdt | update a QR decomposition |
| 2.10 | rotate | Jacobi rotation used by qrupdt |
| 3.3 | splint | polynomial interpolation <br> 3.2 |
| ratint | conic spline interpolation |  |
| 3.3 | spline | rational function interpolation |
| construct a cubic spline |  |  |


| 3.4 | hunt | search a table when calls are correlated |
| :--- | :--- | :--- |
| 3.5 | polcoe | polynomial coefficients from table of values |
| 3.5 | polcof | polynomial coefficients from table of values |
| 3.6 | polin2 | two-dimensional polynomial interpolation |
| 3.6 | bcucof | construct two-dimensional bicubic |
| 3.6 | bcuint | two-dimensional bicubic interpolation |
| 3.6 | splie2 | construct two-dimensional spline |
| 3.6 | splin2 | two-dimensional spline interpolation |
|  |  |  |
| 4.2 | trapzd | trapezoidal rule |
| 4.2 | qtrap | integrate using trapezoidal rule |
| 4.2 | qsimp | integrate using Simpson's rule |
| 4.3 | qromb | integrate using Romberg adaptive method |
| 4.4 | midpnt | extended midpoint rule |
| 4.4 | qromo | integrate using open Romberg adaptive method |
| 4.4 | midinf | integrate a function on a semi-infinite interval |
| 4.4 | midsql | integrate a function with lower square-root singularity |
| 4.4 | midsqu | integrate a function with upper square-root singularity |
| 4.4 | midexp | integrate a function that decreases exponentially |
| 4.5 | qgaus | integrate a function by Gaussian quadratures |
| 4.5 | gauleg | Gauss-Legendre weights and abscissas |
| 4.5 | gaulag | Gauss-Laguerre weights and abscissas |
| 4.5 | gauher | Gauss-Hermite weights and abscissas |
| 4.5 | gaujac | Gauss-Jacobi weights and abscissas |
| 4.5 | gaucof | quadrature weights from orthogonal polynomials |
| 4.5 | orthog | construct nonclassical orthogonal polynomials |
| 4.6 | quad3d | integrate a function over a three-dimensional space |
|  |  |  |
| 5.1 | eulsum | sum a series by Euler-van Wijngaarden algorithm |
| 5.1 | gammln | logarithm of gamma function |
| 6.1 | factrl | factorial function |
| 6.1 | bico | binomial coefficients function |
| 5.3 | factln | logarithm of factorial function |
| 5.3 | poldiv | ratval | | divide one polynomial by another |
| :--- |
| 5.7 |


| 6.1 | beta | beta function |
| :---: | :---: | :---: |
| 6.2 | gammp | incomplete gamma function |
| 6.2 | gammq | complement of incomplete gamma function |
| 6.2 | gser | series used by gammp and gammq |
| 6.2 | gcf | continued fraction used by gammp and gammq |
| 6.2 | erf | error function |
| 6.2 | erfc | complementary error function |
| 6.2 | erfcc | complementary error function, concise routine |
| 6.3 | expint | exponential integral $E_{n}$ |
| 6.3 | ei | exponential integral Ei |
| 6.4 | betai | incomplete beta function |
| 6.4 | betacf | continued fraction used by betai |
| 6.5 | bessj0 | Bessel function $J_{0}$ |
| 6.5 | bessy0 | Bessel function $Y_{0}$ |
| 6.5 | bessj1 | Bessel function $J_{1}$ |
| 6.5 | bessy1 | Bessel function $Y_{1}$ |
| 6.5 | bessy | Bessel function $Y$ of general integer order |
| 6.5 | bessj | Bessel function $J$ of general integer order |
| 6.6 | bessi0 | modified Bessel function $I_{0}$ |
| 6.6 | bessk0 | modified Bessel function $K_{0}$ |
| 6.6 | bessi1 | modified Bessel function $I_{1}$ |
| 6.6 | bessk1 | modified Bessel function $K_{1}$ |
| 6.6 | bessk | modified Bessel function $K$ of integer order |
| 6.6 | bessi | modified Bessel function $I$ of integer order |
| 6.7 | bessjy | Bessel functions of fractional order |
| 6.7 | beschb | Chebyshev expansion used by bessjy |
| 6.7 | bessik | modified Bessel functions of fractional order |
| 6.7 | airy | Airy functions |
| 6.7 | sphbes | spherical Bessel functions $j_{n}$ and $y_{n}$ |
| 6.8 | plgndr | Legendre polynomials, associated (spherical harmonics) |
| 6.9 | frenel | Fresnel integrals $S(x)$ and $C(x)$ |
| 6.9 | cisi | cosine and sine integrals Ci and Si |
| 6.10 | dawson | Dawson's integral |
| 6.11 | rf | Carlson's elliptic integral of the first kind |
| 6.11 | rd | Carlson's elliptic integral of the second kind |
| 6.11 | rj | Carlson's elliptic integral of the third kind |
| 6.11 | rc | Carlson's degenerate elliptic integral |
| 6.11 | ellf | Legendre elliptic integral of the first kind |
| 6.11 | elle | Legendre elliptic integral of the second kind |
| 6.11 | ellpi | Legendre elliptic integral of the third kind |
| 6.11 | sncndn | Jacobian elliptic functions |
| 6.12 | hypgeo | complex hypergeometric function |
| 6.12 | hypser | complex hypergeometric function, series evaluation |
| 6.12 | hypdrv | complex hypergeometric function, derivative of |
| 7.1 | ran0 | random deviate by Park and Miller minimal standard |
| 7.1 | ran1 | random deviate, minimal standard plus shuffle |


| 7.1 | ran2 | random deviate by L＇Ecuyer long period plus shuffle |  |
| :---: | :---: | :---: | :---: |
| 7.1 | ran3 | random deviate by Knuth subtractive method |  |
| 7.2 | expdev | exponential random deviates |  |
| 7.2 | gasdev | normally distributed random deviates |  |
| 7.3 | gamdev | gamma－law distribution random deviates |  |
| 7.3 | poidev | Poisson distributed random deviates |  |
| 7.3 | bnldev | binomial distributed random deviates |  |
| 7.4 | irbit1 | random bit sequence |  |
| 7.4 | irbit2 | random bit sequence | $\stackrel{0}{\text { ¢ }}$ |
| 7.5 | psdes | ＂pseudo－DES＂hashing of 64 bits | 者 0 －${ }^{\circ}$ |
| 7.5 | ran4 | random deviates from DES－like hashing | \％ |
| 7.7 | sobseq | Sobol＇s quasi－random sequence |  |
| 7.8 | vegas | adaptive multidimensional Monte Carlo integration |  |
| 7.8 | rebin | sample rebinning used by vegas | － |
| 7.8 | miser | recursive multidimensional Monte Carlo integration |  |
| 7.8 | ranpt | get random point，used by miser |  |
| 8.1 | piksrt | sort an array by straight insertion |  |
| 8.1 | piksr2 | sort two arrays by straight insertion | N |
| 8.1 | shell | sort an array by Shell＇s method |  |
| 8.2 | sort | sort an array by quicksort method | ＜${ }^{2}$ |
| 8.2 | sort2 | sort two arrays by quicksort method |  |
| 8.3 | hpsort | sort an array by heapsort method |  |
| 8.4 | indexx | construct an index for an array | － |
| 8.4 | sort3 | sort，use an index to sort 3 or more arrays |  |
| 8.4 | rank | construct a rank table for an array | 운ำ完0 |
| 8.5 | select | find the $N$ th largest in an array |  |
| 8.5 | selip | find the $N$ th largest，without altering an array | $\stackrel{\text { O }}{\text { O }}$ |
| 8.5 | hpsel | find $M$ largest values，without altering an array | 를 |
| 8.6 | eclass | determine equivalence classes from list | $\stackrel{\text { O }}{\sim}$ |
| 8.6 | eclazz | determine equivalence classes from procedure |  |
| 9.0 | scrsho | graph a function to search for roots |  |
| 9.1 | zbrac | outward search for brackets on roots |  |
| 9.1 | zbrak | inward search for brackets on roots | 웃 웅 |
| 9.1 | rtbis | find root of a function by bisection |  |
| 9.2 | rtflsp | find root of a function by false－position | \％¢ ¢ ¢ |
| 9.2 | rtsec | find root of a function by secant method |  |
| 9.2 | zriddr | find root of a function by Ridders＇method | 旁号竟 |
| 9.3 | zbrent | find root of a function by Brent＇s method | $\xrightarrow{?}$ |
| 9.4 | rtnewt | find root of a function by Newton－Raphson |  |
| 9.4 | rtsafe | find root of a function by Newton－Raphson and bisection | $\stackrel{\text { ¢ }}{\stackrel{\text { ¢ }}{ }}$ |
| 9.5 | laguer | find a root of a polynomial by Laguerre＇s method |  |
| 9.5 | zroots | roots of a polynomial by Laguerre＇s method with deflation |  |
| 9.5 | zrhqr | roots of a polynomial by eigenvalue methods |  |
| 9.5 | qroot | complex or double root of a polynomial，Bairstow |  |


| 9.6 | mnewt | Newton's method for systems of equations |
| :---: | :---: | :---: |
| 9.7 | lnsrch | search along a line, used by newt |
| 9.7 | newt | globally convergent multi-dimensional Newton's method |
| 9.7 | fdjac | finite-difference Jacobian, used by newt |
| 9.7 | fmin | norm of a vector function, used by newt |
| 9.7 | broydn | secant method for systems of equations |
| 10.1 | mnbrak | bracket the minimum of a function |
| 10.1 | golden | find minimum of a function by golden section search |
| 10.2 | brent | find minimum of a function by Brent's method |
| 10.3 | dbrent | find minimum of a function using derivative information |
| 10.4 | amoeba | minimize in $N$-dimensions by downhill simplex method |
| 10.4 | amotry | evaluate a trial point, used by amoeba |
| 10.5 | powell | minimize in $N$-dimensions by Powell's method |
| 10.5 | linmin | minimum of a function along a ray in N -dimensions |
| 10.5 | f1dim | function used by linmin |
| 10.6 | frprmn | minimize in N -dimensions by conjugate gradient |
| 10.6 | df1dim | alternative function used by linmin |
| 10.7 | dfpmin | minimize in $N$-dimensions by variable metric method |
| 10.8 | simplx | linear programming maximization of a linear function |
| 10.8 | simp1 | linear programming, used by simplx |
| 10.8 | simp2 | linear programming, used by simplx |
| 10.8 | simp3 | linear programming, used by simplx |
| 10.9 | anneal | traveling salesman problem by simulated annealing |
| 10.9 | revcst | cost of a reversal, used by anneal |
| 10.9 | revers | do a reversal, used by anneal |
| 10.9 | trncst | cost of a transposition, used by anneal |
| 10.9 | trnspt | do a transposition, used by anneal |
| 10.9 | metrop | Metropolis algorithm, used by anneal |
| 10.9 | amebsa | simulated annealing in continuous spaces |
| 10.9 | amotsa | evaluate a trial point, used by amebsa |
| 11.1 | jacobi | eigenvalues and eigenvectors of a symmetric matrix |
| 11.1 | eigsrt | eigenvectors, sorts into order by eigenvalue |
| 11.2 | tred2 | Householder reduction of a real, symmetric matrix |
| 11.3 | tqli | eigensolution of a symmetric tridiagonal matrix |
| 11.5 | balanc | balance a nonsymmetric matrix |
| 11.5 | elmhes | reduce a general matrix to Hessenberg form |
| 11.6 | hqr | eigenvalues of a Hessenberg matrix |
| 12.2 | four1 | fast Fourier transform (FFT) in one dimension |
| 12.3 | twofft | fast Fourier transform of two real functions |
| 12.3 | realft | fast Fourier transform of a single real function |
| 12.3 | sinft | fast sine transform |
| 12.3 | cosft1 | fast cosine transform with endpoints |
| 12.3 | cosft2 | "staggered" fast cosine transform |
| 12.4 | fourn | fast Fourier transform in multidimensions |


| 12.5 | rlft3 | FFT of real data in two or three dimensions |
| :---: | :---: | :---: |
| 12.6 | fourfs | FFT for huge data sets on external media |
| 12.6 | fourew | rewind and permute files, used by fourfs |
| 13.1 | convlv | convolution or deconvolution of data using FFT |
| 13.2 | correl | correlation or autocorrelation of data using FFT |
| 13.4 | spctrm | power spectrum estimation using FFT |
| 13.6 | memcof | evaluate maximum entropy (MEM) coefficients |
| 13.6 | fixrts | reflect roots of a polynomial into unit circle |
| 13.6 | predic | linear prediction using MEM coefficients |
| 13.7 | evlmem | power spectral estimation from MEM coefficients |
| 13.8 | period | power spectrum of unevenly sampled data |
| 13.8 | fasper | power spectrum of unevenly sampled larger data sets |
| 13.8 | spread | extirpolate value into array, used by fasper |
| 13.9 | dftcor | compute endpoint corrections for Fourier integrals |
| 13.9 | dftint | high-accuracy Fourier integrals |
| 13.10 | wt1 | one-dimensional discrete wavelet transform |
| 13.10 | daub4 | Daubechies 4-coefficient wavelet filter |
| 13.10 | pwtset | initialize coefficients for pwt |
| 13.10 | pwt | partial wavelet transform |
| 13.10 | wtn | multidimensional discrete wavelet transform |
| 14.1 | moment | calculate moments of a data set |
| 14.2 | ttest | Student's $t$-test for difference of means |
| 14.2 | avevar | calculate mean and variance of a data set |
| 14.2 | tutest | Student's $t$-test for means, case of unequal variances |
| 14.2 | tptest | Student's $t$-test for means, case of paired data |
| 14.2 | ftest | $F$-test for difference of variances |
| 14.3 | chsone | chi-square test for difference between data and model |
| 14.3 | chstwo | chi-square test for difference between two data sets |
| 14.3 | ksone | Kolmogorov-Smirnov test of data against model |
| 14.3 | kstwo | Kolmogorov-Smirnov test between two data sets |
| 14.3 | probks | Kolmogorov-Smirnov probability function |
| 14.4 | cntab1 | contingency table analysis using chi-square |
| 14.4 | cntab2 | contingency table analysis using entropy measure |
| 14.5 | pearsn | Pearson's correlation between two data sets |
| 14.6 | spear | Spearman's rank correlation between two data sets |
| 14.6 | crank | replaces array elements by their rank |
| 14.6 | kendl1 | correlation between two data sets, Kendall's tau |
| 14.6 | kendl2 | contingency table analysis using Kendall's tau |
| 14.7 | ks2d1s | $\mathrm{K}-\mathrm{S}$ test in two dimensions, data vs. model |
| 14.7 | quadct | count points by quadrants, used by ks2d1s |
| 14.7 | quadvl | quadrant probabilities, used by ks2d1s |
| 14.7 | ks2d2s | $\mathrm{K}-\mathrm{S}$ test in two dimensions, data vs. data |
| 14.8 | savgol | Savitzky-Golay smoothing coefficients |
| 15.2 | fit | least-squares fit data to a straight line |


| 15.3 | fitexy | fit data to a straight line, errors in both $x$ and $y$ |
| :--- | :--- | :--- |
| 15.3 | chixy | used by fitexy to calculate a $\chi^{2}$ |
| 15.4 | lfit | general linear least-squares fit by normal equations |
| 15.4 | covsrt | rearrange covariance matrix, used by lfit |
| 15.4 | svdfit | linear least-squares fit by singular value decomposition |
| 15.4 | svdvar | variances from singular value decomposition |
| 15.4 | fpoly | fit a polynomial using lfit or svdfit |
| 15.4 | fleg | fit a Legendre polynomial using lfit or svdfit |
| 15.5 | mrqmin | nonlinear least-squares fit, Marquardt's method |
| 15.5 | mrqcof | used by mrqmin to evaluate coefficients |
| 15.5 | fgauss | fit a sum of Gaussians using mrqmin |
| 15.7 | medfit | fit data to a straight line robustly, least absolute deviation |
| 15.7 | rofunc | fit data robustly, used by medfit |
|  |  |  |
| 16.1 | rk4 | integrate one step of ODEs, fourth-order Runge-Kutta |
| 16.1 | rkdumb | integrate ODEs by fourth-order Runge-Kutta |
| 16.2 | rkqs | integrate one step of ODEs with accuracy monitoring |
| 16.2 | rkck | Cash-Karp-Runge-Kutta step used by rkqs |
| 16.2 | odeint | integrate ODEs with accuracy monitoring |
| 16.3 | mmid | integrate ODEs by modified midpoint method |
| 16.4 | bsstep | integrate ODEs, Bulirsch-Stoer step |
| 16.4 | pzextr | polynomial extrapolation, used by bsstep |
| 16.4 | rzextr | rational function extrapolation, used by bsstep |
| 16.5 | stoerm | integrate conservative second-order ODEs |
| 16.6 | stiff | integrate stiff ODEs by fourth-order Rosenbrock |
| 16.6 | jacobn | sample Jacobian routine for stiff |
| 16.6 | derivs | sample derivatives routine for stiff |
| 16.6 | simpr | integrate stiff ODEs by semi-implicit midpoint rule |
| 16.6 | stifbs | integrate stiff ODEs, Bulirsch-Stoer step |
| 18.1 | fred2 | solve linear Fredholm equations of the second kind |
| 18.1 | fredin | interpolate solutions obtained with fred2 |
| 18.2 | voltra | linear Volterra equations of the second kind |
| 18.3 | wwghts | quadrature weights for an arbitrarily singular kernel |
| 17.3 | kermom | sample routine for moments of a singular kernel |
| 17.2 | quadmx | sample routine for a quadrature matrix |
| 17.3 | shootf | solvde | | solve two point boundary value problem by shooting |
| :--- |
| 17.3 | | two point boundary value problem, solve by relaxation |
| :--- |


| 18.3 | fredex | example of solving a singular Fredholm equation |
| :--- | :--- | :--- |
| 19.5 | sor | elliptic PDE solved by successive overrelaxation method |
| 19.6 | mglin | linear elliptic PDE solved by multigrid method |
| 19.6 | rstrct | half-weighting restriction, used by mglin, mgfas |
| 19.6 | interp | bilinear prolongation, used by mglin, mgfas |
| 19.6 | addint | interpolate and add, used by mglin |
| 19.6 | slvsml | solve on coarsest grid, used by mglin |
| 19.6 | relax | Gauss-Seidel relaxation, used by mglin |
| 19.6 | resid | calculate residual, used by mglin |
| 19.6 | copy | utility used by mglin, mgf as |
| 19.6 | fillo | utility used by mglin |
| 19.6 | maloc | memory allocation utility used by mglin, mgfas |
| 19.6 | mgfas | nonlinear elliptic PDE solved by multigrid method |
| 19.6 | relax2 | Gauss-Seidel relaxation, used by mgfas |
| 19.6 | slvsm2 | solve on coarsest grid, used by mgfas |
| 19.6 | lop | applies nonlinear operator, used by mgfas |
| 19.6 | matadd | utility used by mgfas |
| 19.6 | matsub | utility used by mgfas |
| 19.6 | anorm2 | utility used by mgfas |
|  |  |  |
| 20.1 | machar | diagnose computer's floating arithmetic |
| 20.2 | igray | Gray code and its inverse |
| 20.3 | icrc1 | cyclic redundancy checksum, used by icrc |
| 20.3 | icrc | cyclic redundancy checksum |
| 20.3 | decchk | decimal check digit calculation or verification |
| 20.4 | hufmak | construct a Huffman code |
| 20.4 | hufapp | append bits to a Huffman code, used by hufmak |
| 20.4 | hufenc | use Huffman code to encode and compress a character |
| 20.4 | hufdec | use Huffman code to decode and decompress a character |
| 20.5 | arcmak | construct an arithmetic code |
| 20.5 | arcode | encode or decode a character using arithmetic coding |
| 20.5 | arcsum | add integer to byte string, used by arcode |
| 20.6 | mpops | multiple precision arithmetic, simpler operations |
| 20.6 | mpmul | multiple precision multiply, using FFT methods |
| 20.6 | mpinv | multiple precision reciprocal |
| 20.6 | mpdiv | multiple precision divide and remainder |
| 20.6 | mpsqrt | multiple precision square root |
| 20.6 | mp2dfr | multiple precision conversion to decimal base |
| 20.6 | mppi | multiple precision example, compute many digits of $\pi$ |
| 19 |  |  |

